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Feature Extraction using Dimensionality Reduction Techniques: Capturing the Human Perspective

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FEATURE EXTRACTION USING DIMENSIONALITY REDUCITON TECHNIQUES: CAPTURING THE HUMAN PERSPECTIVE

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Abstract

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The purpose of this paper is to determine if any of the four commonly used dimensionality reduction techniques are reliable at extracting the same features that humans perceive as distinguishable features. The four dimensionality reduction techniques that were used in this experiment were Principal Component Analysis (PCA), Multi-Dimensional Scaling (MDS), Isomap and Kernel Principal Component Analysis (KPCA). These four techniques were applied to a dataset of images that consist of five infrared military vehicles. Out of the four techniques three out of the five resulting dimensions of PCA matched a human feature. One out of five dimensions of MDS matched a human feature. Two out of five dimensions of Isomap matched a human feature. Lastly, none of the resulting dimensions of KPCA matched any of the features that humans listed. Therefore PCA was the most reliable technique for extracting the same features as humans when given a set number of images.

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Introduction

Motivation

In the field of computer vision, computer scientists have strived to make a computer's vision similar to human vision using methods such as neural networks and dimensionality reduction techniques for image processing. In the case of dimensionality reduction the features that are extracted are generally unknown. However if these features were determined to be the same features that humans perceive when given a set of images it will provide a better understanding of dimensionality reduction techniques. This understanding could lead to advancements in the field of pattern recognition and tracking that uses these dimensionality reduction techniques as a way to analyze and/or process images.

Human Perspective

The human brain takes less than a fraction of a second for a human to recognize an object. The part of the brain which is responsible for object recognition is the visual cortex, located in the cerebral cortex. There are over 40 areas in the primary visual cortex, which could explain why the underlying mechanisms for perceiving objects is not well understood [8]. Despite the fact that this area isn't well understood there are different theories on how the brain recognizes objects. The most popular theory is the Two Stream Hypothesis. This hypothesis states that there are two streams, the ventral and the dorsal streams located inside of the primary visual cortex.

In the beginning, the two streams were believe to each have their own separate functions. The ventral stream was thought to be in charge of visual features and identification. While the dorsal stream was thought to be in charge of spatial analysis [7] [8]. However, another model, the perception-action model for the two streams has been proposed. This model states that there is no real separation for the functions of the ventral and the dorsal stream instead, there is a distinct difference between the vision for action and the vision for perception which can be mapped onto the two streams. Both of these concepts have been contradicted which furthers the difficulty in understanding processing of objects in humans. Therefore, the only part that is really understand is that during the process of recognition neurons are activated along the ventral and dorsal streams [19].

Computer Vision

Computers do not recognize objects in the same way that humans recognize objects. However, the field of computer vision is aiming to produce computers that perceive objects similarly to the way humans perceive objects [14]. Therefore, the definition of computer vision is not the way that computers recognize objects, but instead the automatic extraction, analysis and understanding of useful information from images. For grayscale images, computers often view these images as arrays of numbers between 0 and 255. The size of the array multiple images of the same size is the number of images by the length times the width of the images. For examples 5 images that are 40 by 70 pixels would result in an array of size 5 by 3000. Therefore to the computer an image would be perceived by its pixel values instead of the objects in the image.

Introduction to Dimensionality reduction

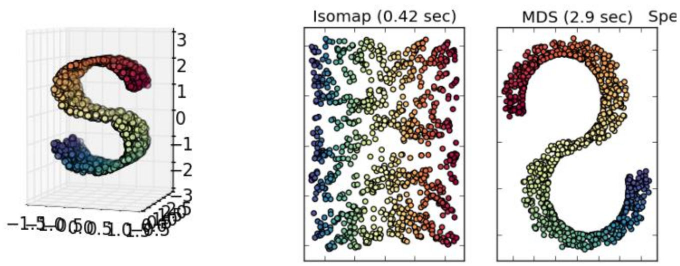


Illustration 1. These pictures, provided by sklearn, show the results of two different dimensionality reduction techniques Isomap, shown in the second image, and MDS, shown in the third image given the data set in the first image.

The main goal of dimensionality reduction techniques is to find a low-dimensional representation of high-dimensional data. This is done with the assumption that in the high-dimensional space there exists a low-dimensional manifold embedded in that space which contains all of the important features. These lower dimensions can be seen as features that were extracted from the higher dimensional space.

These techniques can be separated into two different categories, linear or nonlinear. Linear techniques are used to find a linear mapping from the higher dimension to the lower dimension [4] [5] [6]. How the data is mapped to the lower dimensions is often accomplished with the use of eigenvectors and eigenvalues. These eigenvectors and eigenvalues are a set of vectors that are associated with a system of linear equations, where each vector represents a dimension. For example, when the original data is projected onto the first 3 eigenvectors the original high dimensional data now exists in a 3 dimensional space. Since there isn't always a linear mapping, nonlinear techniques were created to help deal with nonlinear datasets [15]. Real world datasets are usually nonlinear. However, success with linear techniques over nonlinear techniques on real world datasets is possible [4] [5] [6]. Due to the probability that linear techniques will be more successful with the real world data set than nonlinear techniques there was a comparison between the linear and nonlinear results in this experiment.

Problem Statement

The goal of this experiment is to determine if dimensionality reduction technique can extract the same features as humans when given a set of images, despite the difference in how objects are viewed. There were four different dimensionality reduction techniques that were used in this experiment. The four techniques included Principal Component Analysis, also known as PCA, Multi-Dimensional Scaling, also known as MDS, Isomap and Kernel Principal Component Analysis, also known as Kernel PCA. The goal was to be achieved without altering the techniques in any way. Therefore, no filters or training was used on the dataset or the techniques.

Related Works

Dimensionality reduction is often used in the field of computer vision mostly for pattern recognition. There are two reasons that dimensionality reduction is used in pattern recognition: reducing high dimensional data and most importantly, feature extraction. The importance of using dimensionality reduction techniques for feature extraction in pattern recognition is investigated in [11].

Though feature extraction is important in pattern recognition, the features that are extracted are not understood which is why feature templates are used often. In [12] the question regarding if the features that are extracted by computers are the same features that are extracted by humans is asked but never verified. This experiment investigates further into the comparison of computer extracted features and human extracted features in order to verify the question asked in [12]

In [13] they also investigate features that are extracted but only for classification purposes by conventional neural networks. The work in [13] is similar to this experiment as it is investigating features that are extracted by a computer vision method. The difference is the method they used which was trained neural networks while this experiment used dimensionality reduction techniques.

Experiment

Survey Creation

In order to determine what features humans would extract from the dataset, a number of different opinions from different people were needed. In this experiment two surveys were created that were distributed to a number of people from different backgrounds. The first survey was used to gather a list of features that humans extracted from the different vehicles. For this survey, two different methods were compared to determine the best way to ask humans to extract features. The first method was to provide a list of features to be selected in order of initial extraction by humans. The second method was to present humans with the images and ask them to list features from most distinguishable to least distinguishable. The second method was used because it would contain less bias. The trade-off of not giving features was the possibility of having unusable answers.

Q1. Which features help you tell the difference between the five vehicles (please list from most distinguishable to less distinguishable)?

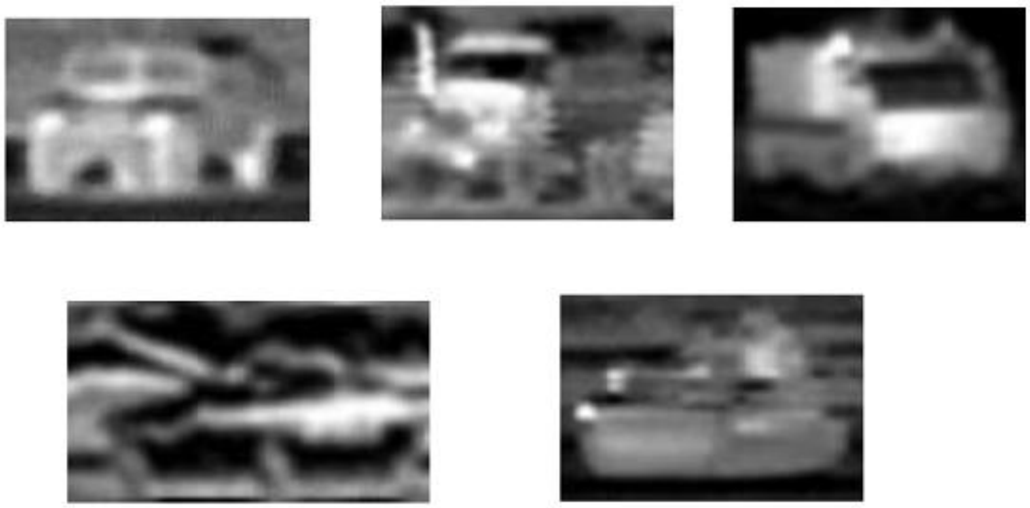
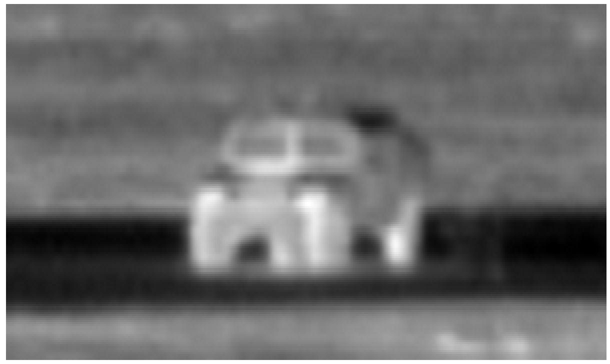


Illustration 2: This image is the question that was posed in the first survey. After looking at these images people were to list features from most distinguishable to least distinguishable.

After the survey was complete, with the help of Qualtrics [16], a survey tool provided by Wright State, it was distributed to twenty people. After gathering the responses of the twenty people, the second survey was created. The purpose of the second survey was to gather people's opinion on how present the top five features given in the first survey was in each vehicle. For the second survey people were presented with questions that asked them to agree or disagree if a feature attribute was present in each vehicle image.

Question 1. For each statement please rate how closely the statement accurately describes each image?

Vehicle 1



	Strongly Agree	Agree	Not Sure	Disagree	Strongly Disagree
The vehicle appears to be on the larger side	○	○	○	○	○
The vehicle appears to have a box like shape	○	○	○	○	○
The vehicle seems to have a large number of windows	○	○	○	○	○
The vehicle appears to have a weapon attached	○	○	○	○	○
The vehicle's windows appear to be square shaped	○	○	○	○	○
The vehicle appears to have a large number of wheels	○	○	○	○	○

Illustration 3: This picture shows the question asked in the second survey. The image only shows the first vehicle image, but the rest of the vehicles are asked the same questions in the same matter.

Dataset

The dataset that was used in this experiment was the Comanche Dataset, which was provided by the United States Army. This dataset consists of images of ten different vehicles with IDs 01 through 10. Each image of a vehicle covers a 9 x 4.5 meter area which is normalized to 75 x 40 pixels. The total number of images for each vehicle is made up of a complete 360 degree rotation around the vehicle in five degree increments (0 to 360 degrees). Each of these images were taken at various ranges (in meters: 2000, 2043, 3000, 3272, 3500) and locations.

Illustration 4. Image taken from the dataset of target 01 from range 2000 at degree 15.

In all, there are 20,742 images, which were taken using a mixture of long wave and forward looking infrared techniques. The Infrared spectrum is a small section in the electromagnetic spectrum, which contains short wavelengths (gamma, x-rays, and ultraviolet) to longer wavelengths (visible light, infrared, and microwaves), that exist after the red end of the visible light spectrum ranging from 0.7 micrometers to 100 micrometers[1]. The thermal region is the part of the infrared spectrum that is used by sensors and ranges from 3 micrometers to 14 micrometers. The area between 3 and 5 micrometers is known as mid wave, and the area between 8 to 14 micrometers is known as long wave and can be detected by sensors [2]. In order to produce images of the long wave thermal radiation emitted by the vehicles in the dataset, forward looking infrared sensing was used.

For this experiment, instead of using all 20,742 only a small portion of these images were used. First, instead of using all of the vehicles only five of the ten targets were used. Also, instead of using a five degree increment around the vehicles only images taken at degrees 10, 15, 20, 25 and 30 were used. Out of the 20,742 images available for this experiment, approximately 700 were used. The targets were chosen based on their diversity in features and vehicle type.

Using a smaller number of images will help with accuracy, time and storage of feature extraction. Better accuracy is achieved by excluding the variance of rotation which would affect the results of PCA. Also, with fewer targets the complexity of the experiment was lessened saving time and storage.

PCA

Principal Component Analysis, also known as PCA, is one of the most popular linear projection dimensionality reduction techniques which was why it was chosen for this experiment. PCA finds the lower dimensional representation of the original data, by finding a linear mapping from the higher dimensional data to a lower dimensional representation. This is accomplished by projecting the original data into a linear subspace while losing as little information as possible. In PCA, information is interpreted as the total amount of variance in the original input variables. Therefore PCA can be seen as a technique that derives a reduced set of linear projection, in descending order by variance, from a collection of variables.

For example, suppose that the input, X is made up of n random vectors of size D. This means that $X = (x_1,\ldots,x_n)$, where x_i, a row in X, is of size D and $i = 1,2,\ldots n$. The variance of the data is obtained in this experiment using the covariance matrix.

Covariance is the measure of how two random variables change together. This means that the covariance matrix of X a (n x D) matrix will be a (D x D) matrix where every position contains the covariance value of one point in X by another point in X. This means that the (i^{th}, j^{th}) position in the covariance matrix for X was the covariance value of x_i and x_j. Also, it is important to note that the (i^{th}, j^{th}) position will equal the (j^{th}, i^{th}) position since the covariance function is a symmetric function. Therefore the covariance matrix is a symmetric matrix. Any symmetrical matrix has a spectral decomposition.

The spectral decomposition for the covariance matrix is: $cov(X) = UOU^T$, where O is the diagonal matrix and U is an orthogonal matrix because all of the columns in U, which will be denoted as u_i, where i = 1,2,…D, are orthogonal. Also, the diagonals in O represent the eigenvalues of the covariance matrix and the columns of U represent the associating eigenvectors of the covariance matrix, which are also known as the principal components of the data.

The eigenvalues also represent the variance in the data. Therefore, in order to maximize the variance in the lower dimensional subspace it is important to sort the eigenvalues and associated eigenvectors in descending order. After the eigenvectors and eigenvalues are sorted, PCA takes the first d principal components where $d \leq D$. For this experiment only five dimensions were used therefore d equals five. These vectors are used to create the linear subspace that the original data is projected onto, in order to create a lower dimensional representation.

To project the data onto the subspace, PCA takes the dot product of the original matrix X by the first d principal components, P, which is a (D x d) matrix. This formula mathematically appears as follows: X' = X ▪ P, where X' is the result of the dimensionality reduction done by PCA. Besides using X, it is possible to use the columns of P, which also represent the features/dimensions of the new lower dimensional representation, to help determine what features make up the new subspace. This is done by visualizing the vector as images the same size as the original data images, like in Figures 4.1-4.5 in the appendix. The following is a summary of the steps of PCA [4] [5] [6] [17] [18]:

1. Calculate the cov(X), the covariance matrix of the original data X with number of dimensions D
2. Solve for the eigenvector and eigenvalues of cov(X)
3. Sort the eigenvector and eigenvalues into descending order to maximize the amount of variance
4. Then project X onto first d eigenvectors where $d \leq D$

MDS

Multi-Dimensional Scaling also known as MDS is another popular linear dimensionality reduction technique. The goal of MDS is to find an underlying lower dimensional manifold that exists within the data with the help of proximities. A proximity is any continuous measurement that can be used to tell how close an entity is to another which doesn't have to be a distance. The proximity can be a subjective rate of similarity (closeness) or dissimilarity. There are two different types of MDS based on which function is used as the proximities.

The two types of MDS are metric and nonmetric MDS. Metric MDS preserves the pairwise distance of every point by using Euclidean distance as the proximity function. Nonmetric MDS uses any other function besides distance as the proximity function. For this experiment metric MDS was used instead of nonmetric MDS.

There is only one difference between PCA and metric MDS, even though it is not rare for them to produce the same results. The difference is that metric MDS preserves the pairwise distance of each point while PCA keeps as much variance as possible. Therefore, the steps to MDS and PCA are similar and the only difference is the matrix created from the original data. PCA starts by creating a covariance matrix while metric MDS starts by creating a distance matrix.

For example, the distance matrix given a random matrix, X = (x_1,…,x_n), where x_i, a row in X, is of size D and i = 1,2,…n, would be a (D x D) matrix where every entry is the Euclidean distance of a point in X by another. The Euclidean distance formula for one dimension is | x – y | = $\sqrt{(x-y)2}$. However, since X has D dimensions the Euclidean distance formula that would be used is, d(x_i,x_j) = $\sqrt{((xi1-xj1)2+(xi2-xj2)2+\cdots+(xiD-xjD)2)}$. Therefore, the (i^{th},j^{th}) position in the distance matrix is the Euclidean distance of x_i and x_j.

Since the Euclidean distance formula is symmetrica,l the distance matrix would also be symmetrical and would therefore have a spectral decomposition. Then just as in PCA, the spectral decomposition of the distance matrix, Dis(X) = UOU^T, can be used to find the eigenvector and eigenvalues of the distance matrix. As stated before the columns of U denoted as u_i where i = 1,…,D, are the eigenvectors of the distance matrix while the diagonals of O are the eigenvalues.

Once the eigenvectors and eigenvalues are calculated, metric MDS will take the first d eigenvectors, where d ≤ D, which will be used to create the underlying linear manifold that metric MDS believes the data lies on. This manifold is created by taking the dot product of the original matrix X by the first d eigenvector, P, which is a (D x d) matrix. This formula mathematically appears as follows: X' = X ▪ P, where X' is the results of the dimensionality reduction done by metric MDS. The steps to MDS are as followed [4] [5] [6] [17] [18]:

1. Calculate the pairwise Euclidean distance matrix Dis(X) of the original data
2. Solve for the eigenvector and eigenvalues of Dis(X)
3. Then project X onto first d eigenvectors

Kernel PCA

Kernel Principal Component Analysis, also known as Kernel PCA, is one of the most popular nonlinear PCA techniques which is why it was used in this experiment. Kernel PCA uses linear PCA after first adding a nonlinear filter in the form of a heat function. The kernel function is used to map the original data with the assumption that it is easier to discover the low dimensional structure in a larger space. Therefore, the kernel function should allow a transformation into a higher dimensional space such that given a dataset X = (x_1,…,x_n), where x_i, a row in X, is of size D and i = 1,2,…n, k(x_i) ∈ H and k: R^x → H. This means that results of the kernel function on X exists in space H such that the kernel function maps the original data to the larger space H.

There are many different kernel functions that can be used on the data by Kernel PCA. These functions include polynomial kernel, linear kernel, sigmoid kernel, and Gaussian kernel. For this experiment the Gaussian kernel function was used as it is the most common kernel function. The Gaussian Kernel function is k(X) = $e^{-gamma * P}$ where P is a (D x D) matrix of the square Euclidean distances of every point by each other and gamma can be any random number, however it is usually assigned the value of 0.5. Therefore the kernel matrix is a matrix where the (i^{th}, j^{th}) position, K(i,j) = $k(x_i,x_j)$, where k is the kernel function.

Once the kernel matrix is calculated it is important to center the data in the matrix because the covariance values span the center of the new dimensional space. After the kernel matrix has been centered Kernel PCA then runs the kernel matrix through linear PCA. The steps to Kernel PCA are as follows [4] [5] [6] [17]:

1. Apply kernel function to data set so $K_{ij} = k(x_i,x_j)$ where k is the kernel function
2. Center the kernel matrix, K in the new higher dimensional space
3. Then run the resulting kernel matrix through PCA

Isomap

Isomap is the nonlinear version of MDS which was why it was used in this experiment. The difference between Isomap and MDS is that MDS is looking for a linear underlying manifold, meaning that if there are any curves or convex regions in the manifold MDS wouldn’t find those values. Isomap on the other hand, uses geodesic distance as opposed to Euclidean distance therefore Isomap will find any curves or convex regions in the underlying manifold.

The geodesic distance is the distance between two points over a manifold which allows for curves, unlike Euclidean distance which only allows for straight lines. Though Isomap will find any curves or convex regions in the manifold, Isomap doesn't do well with any holes in the manifold like metric MDS since distance is a continuous function. Despite this difference between Isomap and MDS, the goal of Isomap is also to preserve pairwise distances.

First Isomap will need to calculate the geodesic distance between every point such that given $X = (x_1,\ldots,x_n)$, where x_i, a row in X, is of size D and $i = 1,2,\ldots n$, a geodesic distance matrix, Dis(X), is created where Dis(i,j) = the geodesic distance between x_i and x_j. In order to calculate the geodesic distance a nearest neighbor graph must be created from the original data X. To create the nearest neighbor graph, a nearest neighbor search is conducted by either selecting an N number of neighbors or by having the neighbors be within a r radius around a point. The graph is constructed by either connecting the K nearest neighbors to each point with weights or connecting a point with all points within a ball of radius r with weights, depending on which method that is chosen. In this experiment, the nearest neighbor graph was created using K nearest neighbors.

Once the nearest neighbor graph G is created, a shortest path algorithm, which will calculate the shortest path between every pair of points of a graph, is used on G in order to calculate the geodesic distance of each point. There are two different shortest path algorithms that can be used to efficiently calculate the geodesic distances. These algorithms are Floyd's shortest path algorithm and Dijkstra's shortest path algorithm. After the geodesic distance matrix has been calculated, Isomap runs this matrix through metric MDS, while skipping the step of calculating the Euclidean distance matrix.

The steps for Isomap are as follows [4] [5] [6] [17]:

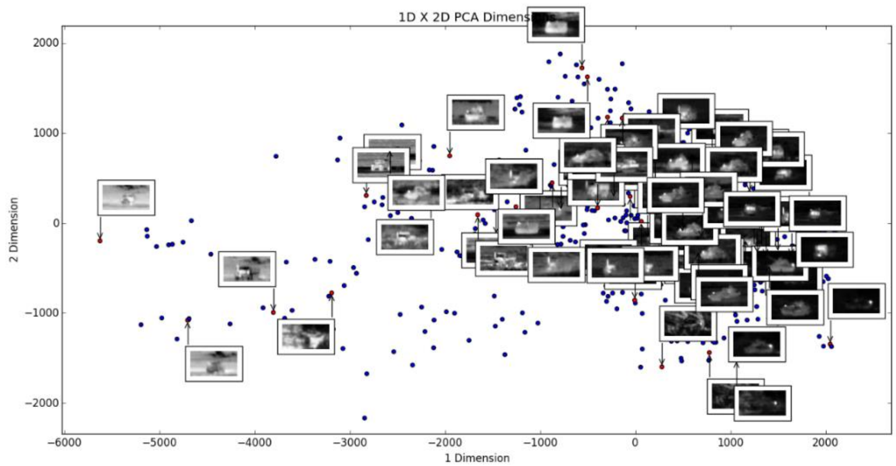
1. Create a nearest neighbor graph, G
2. Calculate the geodesic distance matrix using a shortest path algorithm on G
3. Then run resulting matrix through MDS without calculating the Euclidean distance matrix

Experiment Outline

For this experiment the following steps were followed to help determine whether or not any of the dimensionality reduction techniques were successful in capturing the human perspective. The first step was to figure out the human perspective which was determined by two different surveys distributed to different people. The next step was to run the dataset through the different dimensionality reduction techniques in order to produce each of the first five resulting dimensions for each technique. After the first five dimensions for each technique were retrieved, the next step was to determine which feature each dimension represented.

In order to determine the features that each dimension represents different analyses were used as a means of verification. For this experiment, three different verification analyses were used to verify the different dimensions except in the case of PCA where the eigenvectors were also taken into account. These three verification analyses included plots of the different dimensions, linear visualization of each dimension and 27 different extreme tests on each technique.

The plots include fifteen different graphs for each technique, where every dimension that resulted from a technique was plotted against every other dimension of that same technique (To view plots see appendix under plots).



Plot 1.3. Example of the plot for the resulting first and second dimensions of PCA

The linear visualization analysis was where the first, middle and last 25 images of every dimension were placed in order, by their position in the list of images in the resulting dimension, in a document to help determine what the dimension represented (To view analysis see appendix under Linear Imaging Results).

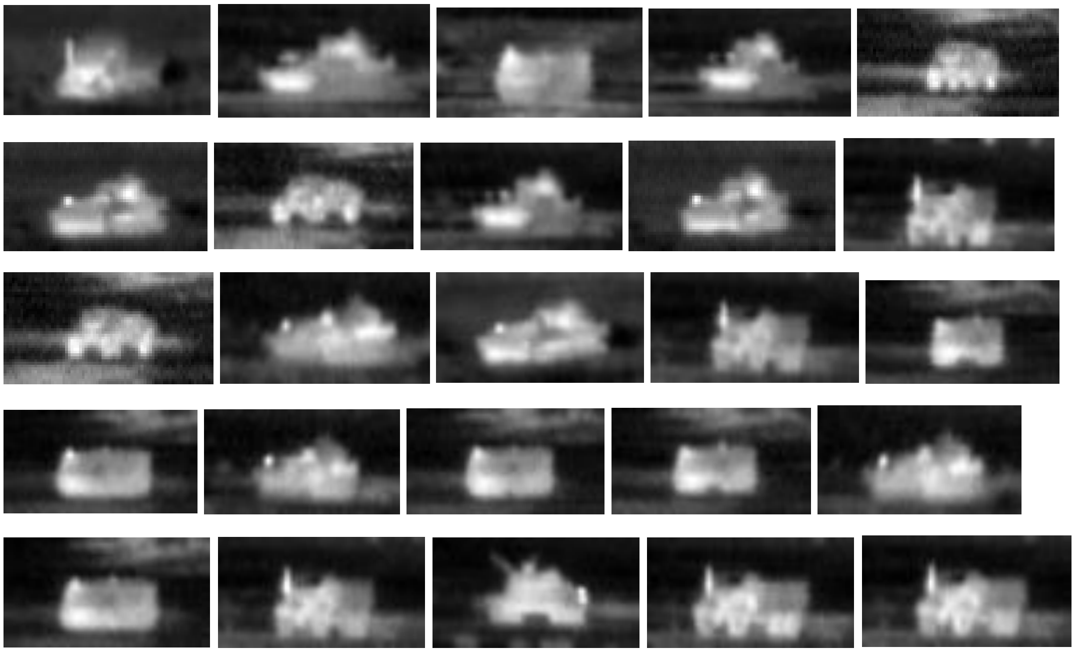


Figure 5.2.3.Partial. Last 25 images of the second dimension of PCA

The extreme analysis was 27 different tests where an associated attribute of one of the features extracted by humans was represented by two images that were new to the techniques. These images represented the extremes of one of the 27 attributes so if the two images appeared near the extremes of the dimension this would prove that the dimension could represent that feature the attribute is associated with (To view results see appendix under Extreme Testing Results).

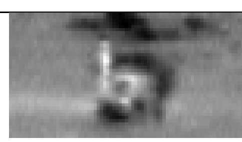
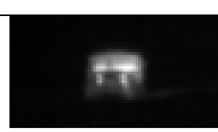
Window Color		672	343	212	663	589
		8	191	153	341	447

Figure 9.3.Partial. This is an example of one of the 27 extreme test. These are the results for the window color extreme test for Isomap.

Result

Survey Results

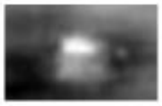
After the survey was taken by 20 people a condensed feature list was made (the original list of features given by each person can be found in the appendix under survey results, figure 1) that combined answers that were similar but worded differently. For the experiment there was the assumption that each dimension could only be related to one feature, therefore only the first five dimensions from each technique would be compared to the human feature list. The human feature list was compiled using the surveyors average feature extraction response using appoint system from most distinguishable to least distinguishable. This was accomplished by assigning a number to each position in the list where the first entry would be given a value of 10, the second entry would be given a value of 9 and so on. Next, a score was given to a feature by summing up the values from every persons' list (the results of this calculation of the rank of each feature can be found in the appendix under survey results, figure 2), though if a feature was listed twice only the highest value was used. After the calculations were completed the features that made up the top 5 out of 9 included wheels, shape, size, presence of weapon, and windows.

Once the top five features were determined, the next survey could be created and distributed. In this survey people were asked how much they agreed that an attribute of one of the top five feature appeared in each vehicle. The results (can view in appendix under survey results, figure 3) of the survey aid in picking which images would be used as extremes in one of the verification analyses. These extremes were chosen based on which vehicle had the majority of people agreeing that the attribute was present and which vehicle had the majority of people disagreeing that the attribute was present.

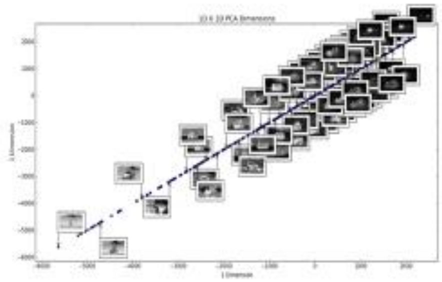
Therefore, with the three verification analyses in place, in order for a dimension to be a confirmed matched with a feature in the human extracted list the dimension would have to pass two of the three verification analyses. If the dimension were to pass one verification analysis the dimension may appear to be one of the features, but it cannot be verified. Therefore, after going through the verification process with all four techniques' results conclusions were drawn on whether any of the dimensions matched or appeared to match any of the features from the human list.

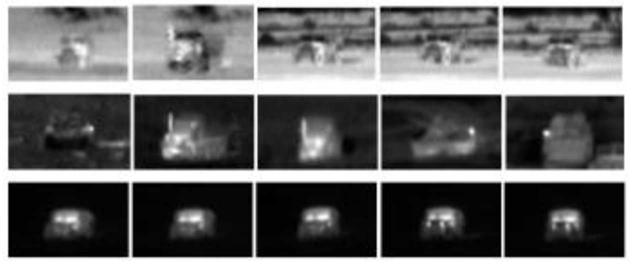
PCA Results

In the case of PCA, the first dimension was represented by window color and windows was ranked five on the list of human extracted features. This conclusion was drawn from the image of the first dimension eigenvector (Figure 4.1 in appendix), the plots of PCA that use the first dimension (In appendix under Plots, under PCA), the linear visualization of the first dimension (Figures 5.1.1, 5.1.2, and 5.1.3 in the appendix) and the results of Window Color test in the Extreme Test (Figure 9.1 in appendix).

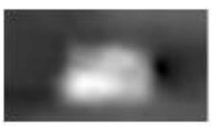


Window Color		18	98	279	457	50
		688	216	344	155	135

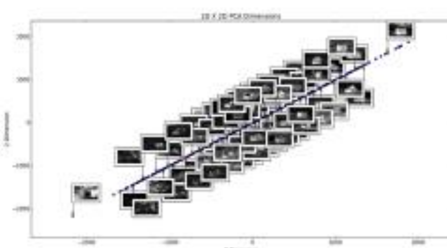


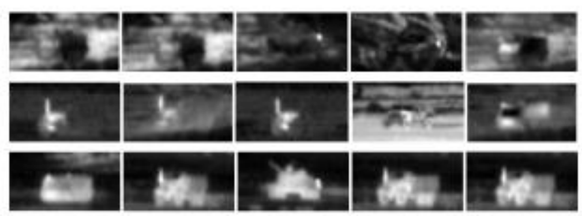


Results 1.1 The images shown here are a collection of a condense version of the verification analysis that confirm that the first dimension of PCA is Window Color. The first image is the eigenvector visualization of the first dimension of PCA. The second image is the results of the extreme test for window color where the third column shows the position of the two extreme images, shown in the second column, in the result of the PCA reduction for the first dimension. The next image is the plot of the results of PCA for the first dimension by itself. The last image shows the first, middle and last five resulting images in the first dimension of PCA.



Color of Vehicle		69	1	30	578	344
		89	639	250	508	388



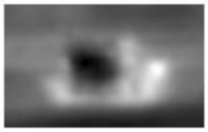


Results 1.2 The images shown here are a collection of a condense version of the verification analysis that confirm that the second dimension of PCA is Color of Vehicle. The first image is the eigenvector visualization of the second dimension of PCA. The second image is the results of the extreme test for color of vehicle where the fourth column shows the position of the two extreme images, shown in the second column, in the result of the PCA reduction for the second dimension. The next image is the plot of the results of PCA for the second dimension by itself. The last image shows the first, middle and last five resulting images in the second dimension of PCA.

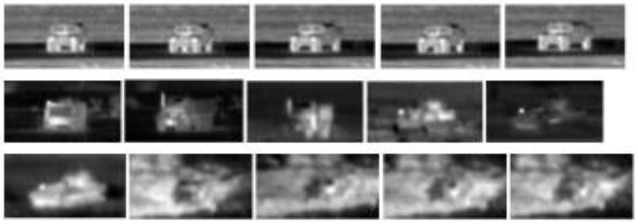
The second dimension of PCA was represented by vehicle color and while it was not in the top five on the human feature list it was ranked 7^{th}. This conclusion was drawn from the image of the second dimension eigenvector (Figure 4.2 in appendix), the plots of PCA, the linear visualization of the second dimension (Figures 5.2.1, 5.2.2. and 5.2.3 in the appendix) and the Color of Vehicle as well as Color Background test in the Extreme test. The third dimension of PCA was undeterminable based on the verification criteria although it appears to represent the ground based on the eigenvector imaging (Figure 4.3 in the appendix). The fourth dimension of PCA was also undeterminable based on the verification criteria, although this dimension appeared to represent the color of the front of the vehicle based on the eigenvector imaging (Figure 4.4 found in the appendix).

Result 1.3 These two images show the eigenvector visualization of the third and fourth resulting dimensions of PCA. They don't confirm that the third dimension is color of ground and the fourth dimension is the color of the front of the vehicle since it is only one of the verification analysis and not two.

The last dimension of PCA was represented by wheel shape and wheels were ranked number one on the list of human extracted features. This conclusion was drawn from the image of the fifth dimension eigenvector (Figure 4.5 in appendix), the linear visualization of the fifth dimension (Figures 5.5.1, 5.5.2 and 5.2.3 in appendix) and the Wheel Shape test in the extreme test.



Wheel Shape		15	247	685	547	18
		68	541	43	6	666



Results 1.4 The images shown here are a collection of a condense version of the verification analysis that confirm that the fifth dimension of PCA is Wheel Shape. The first image is the eigenvector visualization of the fifth dimension of PCA. The second image is the results of the extreme test for wheel shape where the last column shows the position of the two extreme images, shown in the second column, in the result of the PCA reduction for the fifth dimension. The last image shows the first, middle and last five resulting images in the fifth dimension of PCA.

Upon analysis of the verification, the wheel shape test isn't so much the shape of the wheels but the gaps in the wheels based on the color change around the wheels. Therefore, the results for PCA are as follows, the first dimension represents window color, the second dimension represents color of vehicle, the third and fourth dimension are indeterminable and the fifth dimension represents wheel gap. PCA is known for using the maximum amount of variance to create the resulting dimensions, so the first dimension would represent the feature with the most variance and so on. Therefore, these results suggests that the images vary the most in window color, followed by the color intensity of the vehicle with gaps in the wheels being the fifth most varying feature.

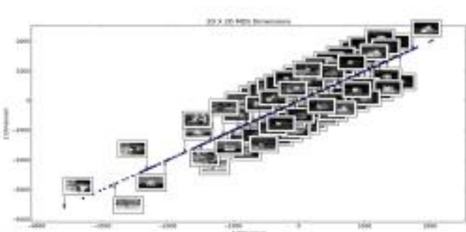
Dimension	1	2	3	4	5
Results	Window Color	Vehicle Color	*Ground	*Front of Vehicle	Gaps in Wheels

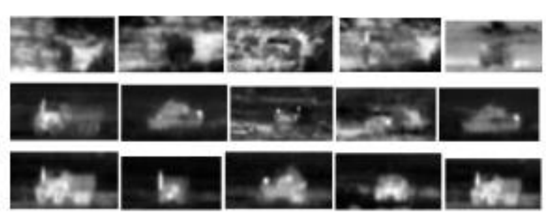
Result 1.5 This chart summarizes the results for PCA. The results with an asterisk in the front are only suggested representations of the dimensions that couldn't be fully determined.

MDS Results

For MDS the results were remarkably different than PCA's. The first dimension of MDS was undeterminable based on the verification criteria and could not be determined through any of the verification tests. The second dimension of MDS was represented by the contrast between the color of the vehicle versus the color of the background. While this color contrast was not a feature on the list of human extracted features, color intensity did place 7th on the list. The conclusion drawn about the second dimension was based on the Color Background test in the extreme test (Figures 9.2 in appendix), the linear visualization (Figures 6.2.1, 6.2.2 and 6.2.3 in appendix) and the plots (In appendix under Plots, under MDS).

Color Background		399	32	9	409	4
		532	636	352	209	101

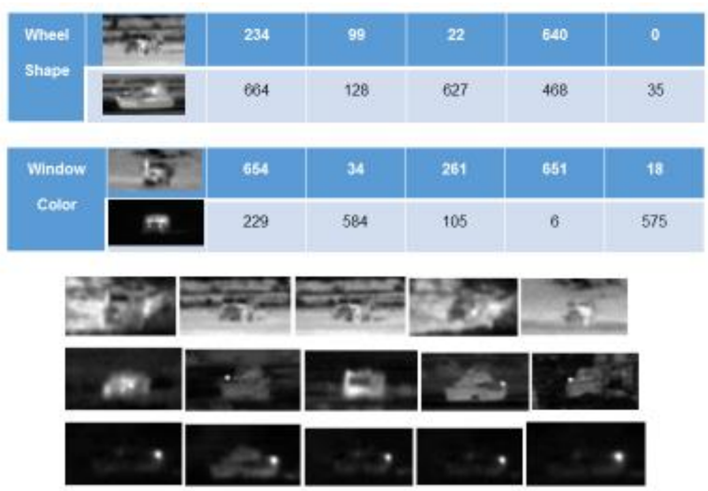




Results 2.1 The images shown here are a collection of a condense version of the verification analysis that confirm that the second dimension of MDS is the contrast between the background color and vehicle color. The first image is the results of the extreme test for Color of Background where the fourth column shows the position of the two extreme images, shown in the

second column, in the result of the MDS reduction for the second dimension. The next image is the plot of the results of MDS for the second dimension by itself. The last image shows the first, middle and last five resulting images in the second dimension of MDS.

The third dimension of MDS was undeterminable based on the verification criteria but it appeared to represent wheel shape based on the Wheel Shape test in the extreme test, which, as mentioned, used the gaps in the wheels based on changes in color in the wheel area. Also the fourth and fifth dimension of MDS was undeterminable based on the verification criteria but the fourth dimension appeared to represent window color, based on the Window Color test in the extreme test, and the fifth dimension overall color intensity, based on the linear visualization (Figures 6.5.1, 6.5.2 and 6.5.3 in appendix).



Wheel Shape		234	99	22	640	0
		664	128	627	468	35

Window Color		654	34	261	651	18
		229	584	105	6	575

Results 2.2 The images shown here are a collection of a condense version of the verification analysis that gives a suggestion to what the last three dimension of MDS represent though it is not determinable. The first image is the results of the extreme test for Wheel Shape where the fifth column shows the position of the two extreme images, shown in the second column, in the result of the MDS reduction for the third dimension. The next image is the results of the extreme test for Window Color where the sixth column shows the position of the two extreme images, shown in the second column, in the result of the MDS reduction for the fourth dimension. The last image shows the first, middle and last five resulting images in the fifth dimension of MDS.

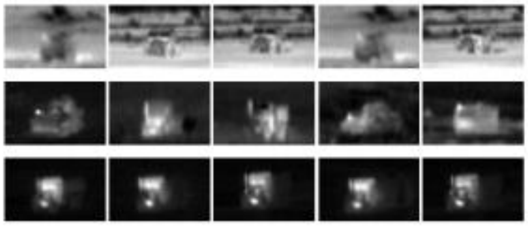
Therefore the results for MDS are as follows: the first dimension was indeterminable, the second dimension was the contrast between the color of the background and color of the vehicle, and the last three dimensions are indeterminable. However, since MDS is known for preserving the pairwise distance of each pixel. This brings about a visual representation of the pattern of similarities among the dataset. Therefore, these results state that in the dataset, the color contrast between the background and the vehicle is one of the similarities between the images in the dataset.

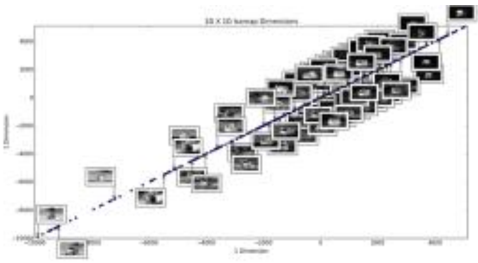
Dimension	1	2	3	4	5
Results	No Match	Color of Vehicle versus Color of Background	*Wheel Shape	*Window Color	*Overall Color Intensity

Result 2.3 This chart summarizes the results for MDS. The results with an asterisk in the front are only suggested representations of the dimensions that couldn't be fully determined.

Isomap Results

For Isomap, the results were more closely aligned with the human extraction list of features than MDS, however not as closely aligned as in PCA. The first dimension of Isomap was represented by window color. This conclusion was drawn from the plots of Isomap that use the first dimension (In appendix under Plots, under PCA), the linear visualization of the first dimension (Figures 7.1.1, 7.1.2, and 7.1.3 in the appendix) and the results of Window Color test in the Extreme Test (Figure 9.3 in appendix).

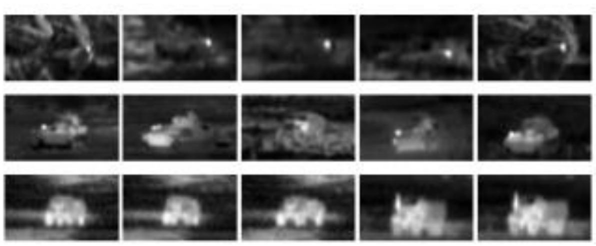


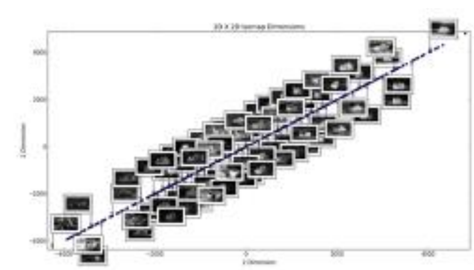


Window Color		672	343	212	663	589
		8	191	153	341	447

Results 3.1 The images shown here are a collection of a condense version of the verification analysis that confirm that the first dimension of Isomap is Window Color. The first image shows the first, middle and last five resulting images in the first dimension of Isomap. The next image is the plot of the results of Isomap for the first dimension by itself. The last image is the results of the extreme test for window color where the third column shows the position of the two extreme images, shown in the second column, in the result of the Isomap reduction for the first dimension.

The second dimension of Isomap also matched a feature from the human extracted list as it represented the color of the vehicle. This conclusion was drawn from the plots of Isomap and the linear visualization of the second dimension (Figures 7.2.1, 7.2.2. and 7.2.3 in the appendix).





Results 3.2 The images shown here are a collection of a condense version of the verification analysis that confirm that the second dimension of Isomap represents Vehicle Color. The first image shows the first, middle and last five resulting images in the second dimension of Isomap. The next image is the plot of the results of Isomap for the second dimension by itself.

The last three dimensions of Isomap were undeterminable based on the verification criteria.

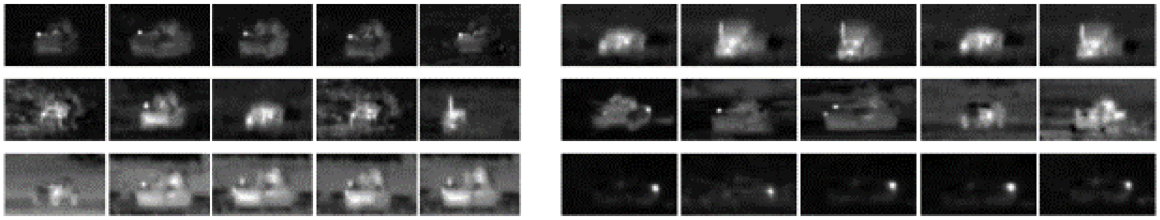
In conclusion, the results for Isomap was as followed, the first dimension represents window color, the second dimension represents vehicle color and the last three dimensions were indeterminable. Isomap is the non-linear version of MDS, where the difference is that instead of preserving the pairwise distances like MDS, Isomap preserves the geodesic distances. This suggests that window color could exists on a curve in the lower dimensional manifold which would explain why window color wasn't a dimension in MDS and color of vehicle was. Therefore, these results state that two similarities in the dataset are window color and vehicle color.

Dimension	1	2	3	4	5
Results	Window Color	Color of Vehicle	No Match	No Match	No Match

Result 3.3 This chart summarizes the results for Isomap.

Kernel PCA Results

Kernel PCA was the least similar to the human list of extracted features. The first two dimensions of Kernel PCA were undeterminable by the verification criteria. However, the first dimension appeared to represent overall color intensity, based on the linear visualization (Figures 8.1.1, 8.1.2, and 8.1.3 in appendix). The second dimension appeared to represent the color of vehicle, based on the linear visualization (Figures 8.2.1, 8.2.2, and 8.2.3 in appendix).



Results 4.1 The images shown here are a collection of a condense version of the verification analysis that gives a suggestion to what the first two dimension of MDS represent though it is not determinable. The first image shows the first, middle and last five resulting images in the first dimension of Kernel PCA. The next image shows the first, middle and last five resulting images in the second dimension of Kernel PCA.

The last three dimensions of Kernel PCA were undeterminable by the verification criteria.

In conclusion, the results for Kernel PCA were that none of the dimensions were determinable based on the verification analysis. This means that the transformation to the higher dimension by the use of a kernel function, hindered the discovery of human extracted feature dimensions, or that the Gaussian kernel function was a poor choice of kernel functions for the dataset.

Dimension	1	2	3	4	5
Results	*Overall Color Intensity	*Color of Vehicle	No Match	No Match	No Match

Result 4.2 This chart summarizes the results for Kernel PCA. The results with an asterisk in the front are only suggested representations of the dimensions that couldn't be fully determined.

Summary of Results

In summary, PCA extracted three human extracted features. These features include window color as the first dimension, vehicle as the second dimension and gaps in wheels based on color intensity as the fifth dimension. MDS only extracted one human extracted feature which was the contrast between the color of the vehicle and the color of the background as the second dimension. Isomap extracted two human extracted features. These features include window color as the first dimension and vehicle color as the second dimension. Lastly, Kernel PCA didn't extract any human extracted features.

In conclusion looking at all of the results the identified dimensions revolve around color, which aligns with the computer reading images as an array of pixel values. In order to get around this, the addition of filters to distinguish shapes can be used.

Technique	First Dimension	Second Dimension	Third Dimension	Fourth Dimension	Fifth Dimension
PCA	Window Color	Vehicle Color	No Match	No Match	Gaps in Wheels
MDS	No Match	Color of Vehicle Compare to Background	No Match	No Match	No Match
Isomap	Window Color	Vehicle Color	No Match	No Match	No Match
Kernel PCA	No Match	No Match	No Match	No Match	No Match

Table 1. This table shows the results of all the dimensions across all four techniques.

Conclusion

In the case of the linear techniques, PCA had three out of its five dimensions match up with one of the features in the human extracted list, while MDS only had one of its five dimensions match one of the features in the human extracted list. Therefore PCA clearly matched the closest with human extracted features in the case of the linear techniques.

For the non-linear techniques Isomap had two out of its five dimensions match up with one of the features in the human extracted list. While Kernel PCA did not have any dimensions that matched up with features in the human extracted list. Therefore Isomap matched the closest with human extracted features in the case of the non-linear techniques.

When it comes to linear versus non-linear techniques, comparing the two is more difficult. In the case of PCA versus its non-linear version Kernel PCA, PCA was able to match three features while Kernel PCA could not match any features. Therefore the linear case was more reliable. In the case of MDS verses its non-linear version Isomap, Isomap was able to match two features while MDS was only able to match one. The nonlinear case appears to be more reliable. However, if you look at linear versus nonlinear overall, PCA and Isomap match the same features in the first two dimensions. PCA was also able to match another feature from the human list. This means that overall, PCA, a linear technique was more reliable at matching some of the features that humans extracted. These results are surprising since the dataset is nonlinear, so it was believed that the nonlinear techniques would do better than the linear techniques.

In conclusion, it appears that PCA, isomap and MDS are capable of extracting the same features as humans but they are not 100% reliable. The most reliable at extracting the same features as humans was PCA since three out of the five dimensions did match a human extracted feature. In second place came Isomap with two out of five dimensions, and lastly MDS with only one of the five dimensions.

In the future adjustments can be made in order to improve and expand upon this experiment. One of the adjustments to be made would be to use a different dataset. Humans see the world using the visual light spectrum and the dataset was collected using the infrared spectrum which is not the spectrum humans see in. This would have had a big influence on the results. Also, the resolution and angle of the images made the images difficult to distinguish, which would have also played a significant role in the accuracy of the results. Another adjustment that could be made would be to edit the feature extraction survey to ask the people to be more specific on which attribute of a feature they are taking from the photo. For example, instead of just saying ‘window’ the subjects would have to say ‘window number’ or ‘window shape’. This will allow for more accurate results. Another way to expand on the experiment is to use different dimensionality reduction techniques to see if any will be more reliable at extracting human features than the ones used in this experiment. One of the last adjustments that can be made to the experiment is the addition of filters which will allow for the extracting of shapes in images, such that the results don’t all revolve around pixel values.

Therefore, this experiment is only a stepping stone to understanding dimensionality reduction techniques and their ability to extract the human's perspective, bringing about advancements in pattern recognition and/or tracking.

Bibliography

1. Tutorial: Fundamentals of Remote Sensing. (n.d.). from http://www.nrcan.gc.ca/earth-sciences/geomatics/satellite-imagery-air-photos/satellite-imagery-products/educational-resources/9309
2. Thermal Remore Sensing. (n.d.). from aulavirtual.agro.unlp.edu.ar/pluginfile.php/19547/mod_folder/content/0/thermal RS.pdf
3. Driggers, R. (2003). *Encyclopedia of optical engineering* (p. 560). New York: Marcel Dekker.
4. Van den Herik, H., Van der Maaten, L., & Postma, E. (2009). Dimensionality Reduction: A Comparative Review. *Journal of Machine Learning Research, 10*, 66-71.
5. Ma, Y. (2012). *Manifold learning theory and applications* (pp. 1-20, 30-32). Boca Raton, Fla.: CRC ;.
6. Ghodsi, A. (2006). Dimensionality Reduction: A Short Tutorial. *Department of Statisticsand Actuarial Science, University of Waterloo,* 1-16.
7. Gril-Spector, K. (2003). The Neural Basis of Object Perception. *Current Opinion in Neurobiology,* 1-8. doi:10.1016/S0959-4388(03)00040-0
8. Thomas Schenk & Robert D. McIntosh (2010*)* Do we have independent visual streams for perception and action?, *Cognitive Neuroscience*, 1:1, 52-62, DOI: 10.1080/17588920903388950
9. Scikit-learn: Machine Learning in Python, Pedregosa *et al.*, JMLR 12, pp. 2825-2830, 2011.

10. About the BMVA. (n.d.). Retrieved from http://www.bmva.org/visionoverview

11. Wang, X., & Paliwal, K. (n.d.). Feature extraction and dimensionality reduction algorithms and their applications in vowel recognition. *Pattern Recognition,* 2429-2439.

12. Turk, M., & Pentland, A. (1991). Face recognition using eigenfaces. *Proceedings. 1991 IEEE Computer Society Conference on Computer Vision and Pattern Recognition*.

13. Zeiler, M., & Fergus, R. (n.d.). Visualizing and Understanding Convolutional Networks. *Computer Vision – ECCV 2014 Lecture Notes in Computer Science,* 818-833.

14. Klette, R. (n.d.). *Concise computer vision: An introduction into theory and algorithms*.

15. Eigenvector. (n.d.). Retrieved from http://mathworld.wolfram.com/Eigenvector.html

16. Qualtrics: Online Survey Software & Insight Platform. (n.d.). Retrieved from http://www.qualtrics.com/

17. Burges, C. (n.d.). Dimension Reduction: A Guided Tour. FNT in Machine Learning Foundations and Trends® in Machine Learning, 275-364.

18. Geyer C. Principal Components Theory Notes. 2007

19. Ungerleider, L., & Haxby, J. (n.d.). 'What' and 'where' in the human brain. Current Opinion in Neurobiology, 157-165.

Appendix

Survey Results

Feature Extraction

1	2	3	4	5	6	7	8	9	10
size	wheels	shape	cannon	headlights					
size	shape	number of wheels	head lights	barrel					
size	shape	number of wheels	headlights	presence of barrel					
Size	Shape	Windows	Exhaust	Color intensity	Presence of Cannon	Wheels & Tracks	Front Ends	Rear Ends	Weights
types of vehicles	type of wheels	types of features attache	lights	engine types	sizes of vehicles	area where they are			
size	windows	shape	wheels	color					
gun	turret	caterpillar tracks	armor						
presence of barrel	number of windows	shape of vehicle	vertical exhaust	presence of wheels					
Headlights	Windshield								
size of vehicle	type wheels	shape of front	tank or truck	lights	windows	number of wheels	shape of back	color (more white or blac	weapons
Shape of vehicle	Window frame shap	Extensions	Realtive size of vehicle	Head lights visible					
Shape	Brightness	Wheels							
shape	shading	size							
type of wheels	barrel								
shape of the front part	presence of turret	shape of wheels							
width	height	light versus dark areas	size of windshield	turret on the tank	smokestack on the truck				
shape	contours	wheels	lights						
Shape	Size	Color	Number of Wheels	Which one has an elephant tru	Which one looks like a tractor				
windshields	cannons	wheels	turret	treads	exhaust pipe	lights	shading showing depth		
no circles for tires	windshields	headlights							

FIGURE 1. This chart shows the features that all twenty people extracted from the five different vehicles.

Feature	Score
Wheel	121
Shape	119
Size	99
Gun	78
Window	76
Headlights	68
Color Intensity	54
Exhaust	25
Type of Vehicle	17
Extensions	8

Figure 2. This is the condense list of features and their score from the ranking calculations.

Feature Labeling

	Question	Strongly Agree	Agree	Not Sure	Disagree	Strongly Disagree	Total Responses	Mean
1	The vehicle appears to be on the larger side	3	14	7	6	0	30	2.53
2	The vehicle appears to have a box like shape	9	16	0	4	1	30	2.07
3	The vehicle seems to have a large number of windows	1	8	7	13	1	30	3.17
4	The vehicle appears to have a weapon attached	0	1	7	15	7	30	3.93
5	The vehicle's windows appear to be square shaped	6	10	5	7	2	30	2.63
6	The vehicle appears to have a large number of wheels	2	1	6	16	5	30	3.7
7	The vehicle appears to have a more of an oval shape	2	2	2	15	9	30	3.9

Figure 3.1. This is the results for the feature labeling survey for vehicle one after thirty people responded.

#	Question	Strongly Agree	Agree	Not Sure	Disagree	Strongly Disagree	Total Responses	Mean
1	The vehicle appears to be on the larger side	10	15	3	1	1	30	1.93
2	The vehicle appears to have a box like shape	3	9	3	12	3	30	3.1
3	The vehicle seems to have a large number of windows	1	0	5	10	14	30	4.2
4	The vehicle appears to have a weapon attached	22	5	1	1	1	30	1.47
5	The vehicle's windows appear to be square shaped	0	3	9	9	9	30	3.8
6	The vehicle appears to have a large number of wheels	7	13	4	1	5	30	2.47
7	The vehicle appears to have a more of an oval shape	2	14	3	8	3	30	2.87

Figure 3.2. This is the results for the feature labeling survey for vehicle two after thirty people responded.

	Question	Strongly Agree	Agree	Not Sure	Disagree	Strongly Disagree	Total Responses	Mean
1	The vehicle appears to be on the larger side	3	19	4	3	1	30	2.33
2	The vehicle appears to have a box like shape	4	14	3	7	2	30	2.63
3	The vehicle seems to have a large number of windows	1	10	5	12	2	30	3.13
4	The vehicle appears to have a weapon attached	1	1	8	15	5	30	3.73
5	The vehicle's windows appear to be square shaped	4	23	0	3	0	30	2.07
6	The vehicle appears to have a large number of wheels	5	16	4	5	0	30	2.3
7	The vehicle appears to have a more of an oval shape	0	1	1	18	10	30	4.23

Figure 3.3. This is the results for the feature labeling survey for vehicle three after thirty people responded.

#	Question	Strongly Agree	Agree	Not Sure	Disagree	Strongly Disagree	Total Responses	Mean
1	The vehicle appears to be on the larger side	10	14	5	1	0	30	1.9
2	The vehicle appears to have a box like shape	3	10	5	9	3	30	2.97
3	The vehicle seems to have a large number of windows	1	3	6	10	10	30	3.83
4	The vehicle appears to have a weapon attached	2	10	9	8	1	30	2.87
5	The vehicle's windows appear to be square shaped	0	5	11	10	4	30	3.43
6	The vehicle appears to have a large number of wheels	2	5	7	6	10	30	3.57
7	The vehicle appears to have a more of an oval shape	3	7	5	12	3	30	3.17

Figure 3.4. This is the results for the feature labeling survey for vehicle four after thirty people responded.

#	Question	Strongly Agree	Agree	Not Sure	Disagree	Strongly Disagree	Total Responses	Mean
1	The vehicle appears to be on the larger side	4	17	4	5	0	30	2.33
2	The vehicle appears to have a box like shape	16	12	1	1	0	30	1.57
3	The vehicle seems to have a large number of windows	0	4	5	16	5	30	3.73
4	The vehicle appears to have a weapon attached	0	7	8	12	3	30	3.37
5	The vehicle's windows appear to be square shaped	7	20	3	0	0	30	1.87
6	The vehicle appears to have a large number of wheels	3	11	10	6	0	30	2.63
7	The vehicle appears to have a more of an oval shape	0	3	2	14	11	30	4.1

Figure 3.5. This is the results for the feature labeling survey for vehicle five after thirty people responded.

Eigenvector Visualization

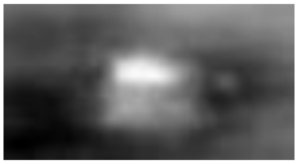


Figure 4.1. The eigenvector image of the resulting first dimension of PCA

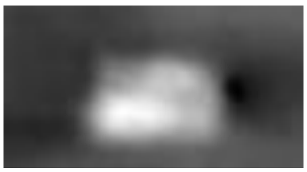


Figure 4.2. The eigenvector image of the resulting second dimension of PCA

Figure 4.3. The eigenvector image of the resulting third dimension of PCA

Figure 4.4. The eigenvector image of the resulting fourth dimension of PCA

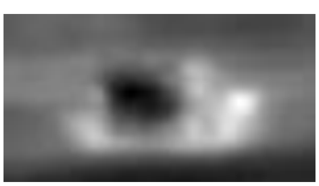
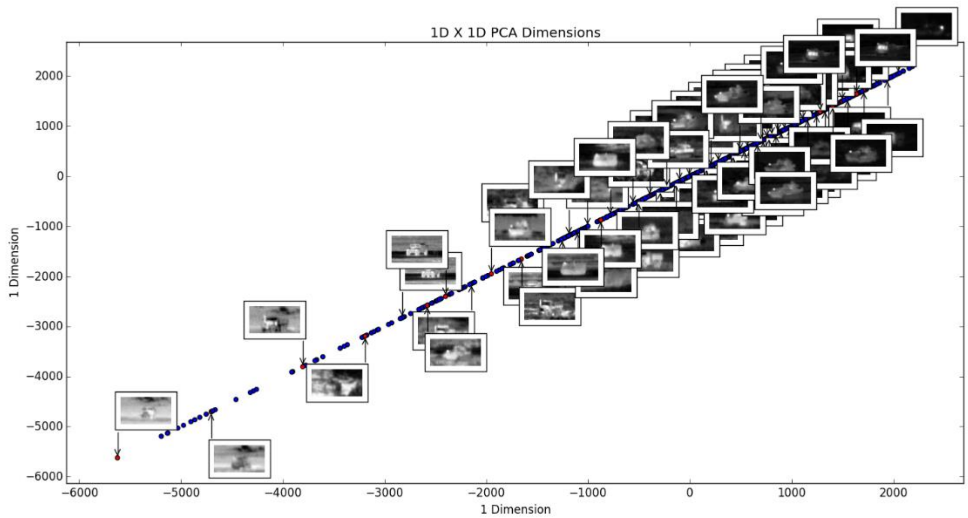


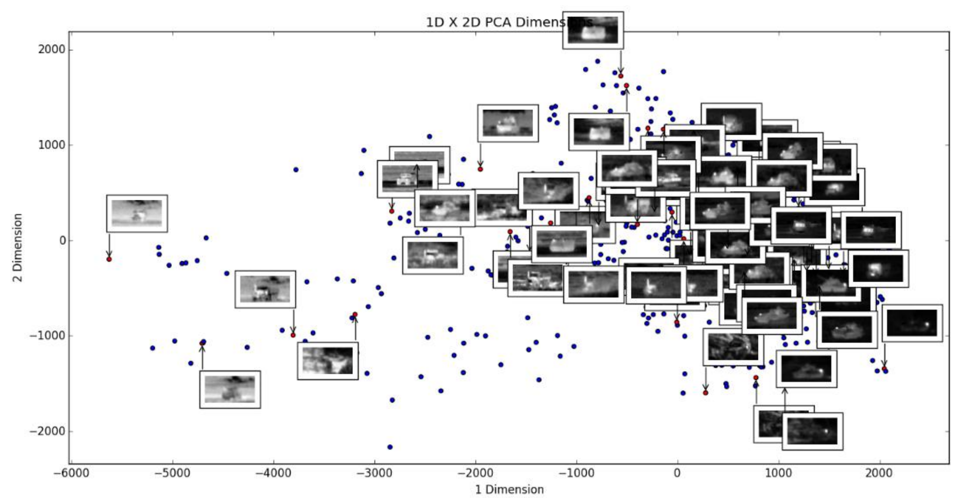
Figure 4.5. The eigenvector image of the resulting fifth dimension of PCA

Plots

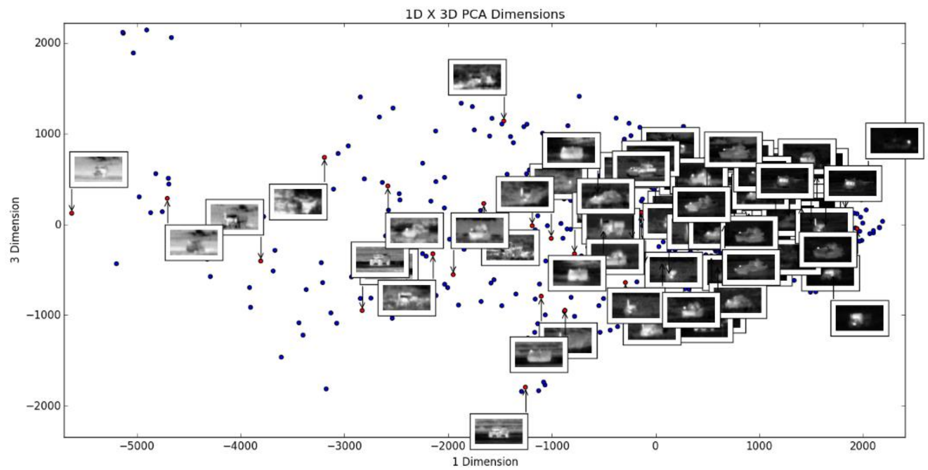
PCA



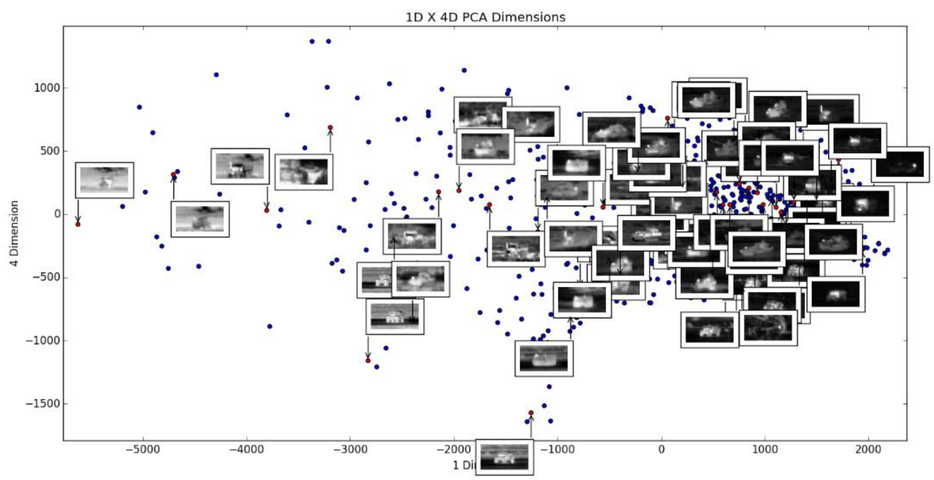
Plot 1.1: Is the plot of the first resulting of PCA dimension by the first resulting of PCA dimension



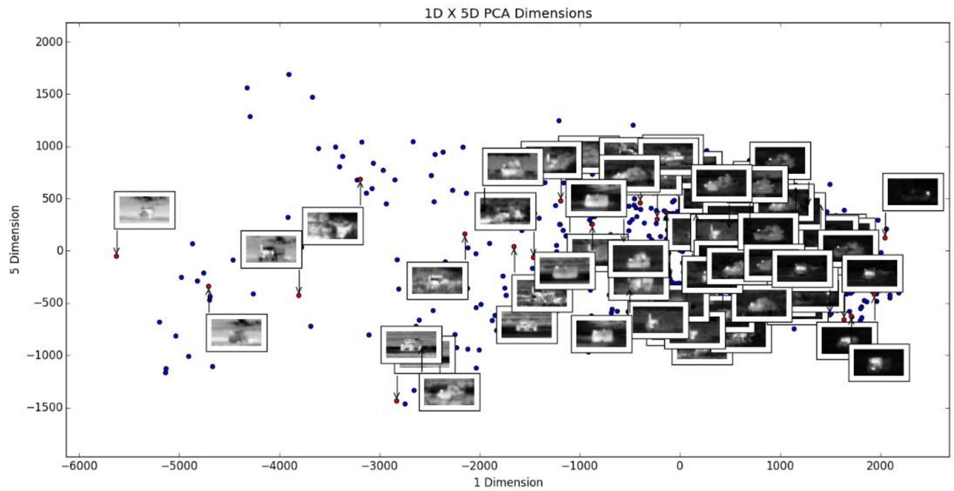
Plot 1.2: Is the plot of the first resulting of PCA dimension by the second resulting of PCA dimension



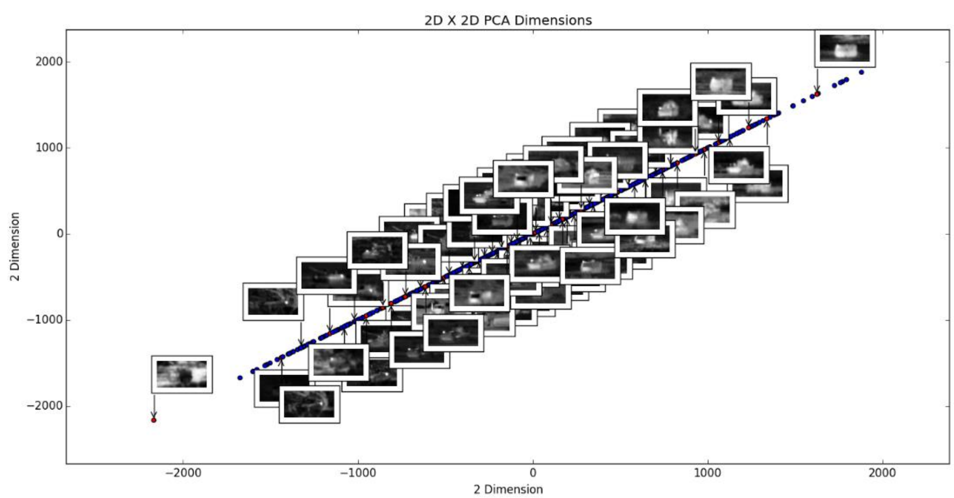
Plot 1.3: Is the plot of the first resulting of PCA dimension by the third resulting of PCA dimension



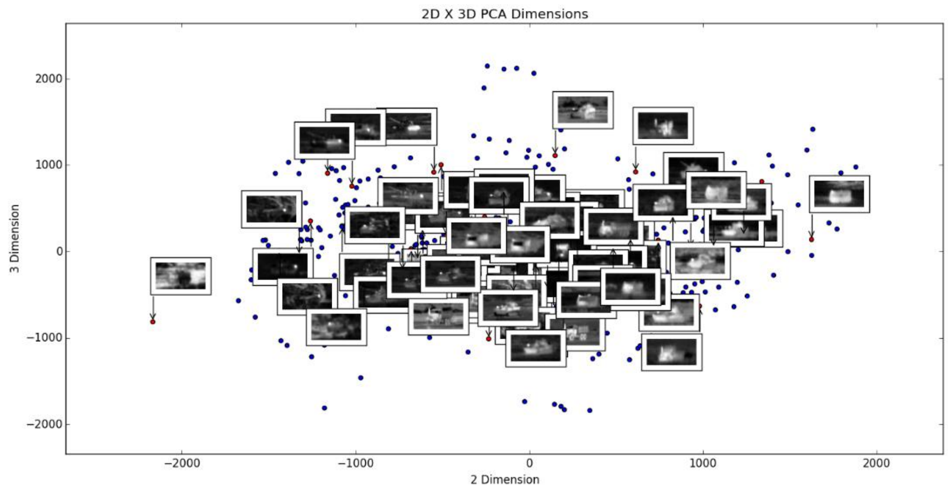
Plot 1.4: Is the plot of the first resulting of PCA dimension by the fourth resulting of PCA dimension



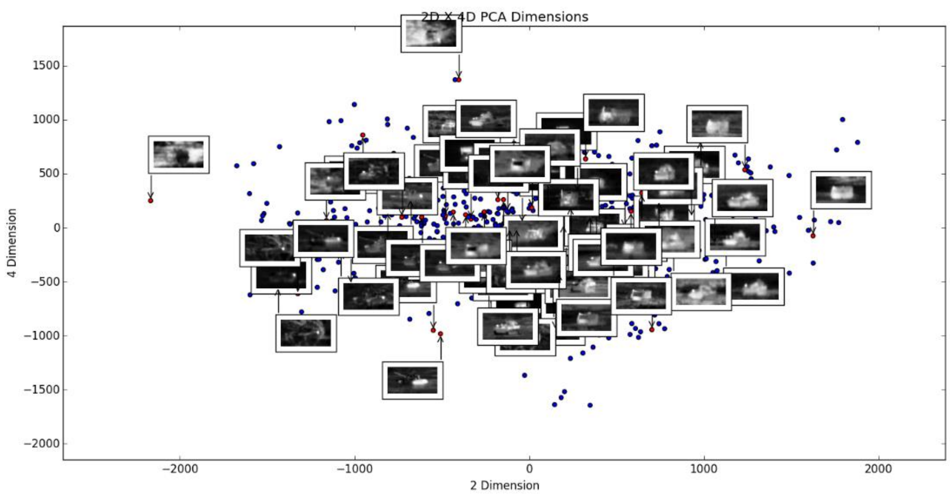
Plot 1.5: Is the plot of the first resulting of PCA dimension by the fifth resulting of PCA dimension



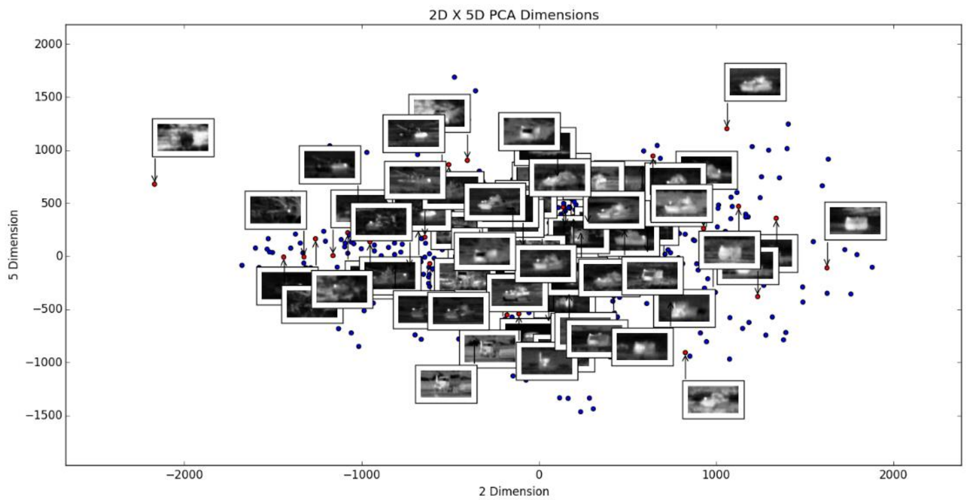
Plot 1.6: Is the plot of the second resulting of PCA dimension by the second resulting of PCA dimension



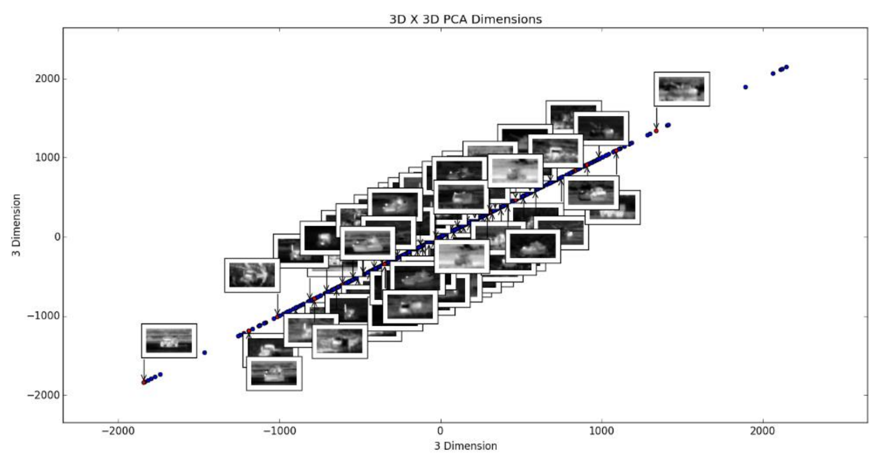
Plot 1.7: Is the plot of the second resulting of PCA dimension by the third resulting of PCA dimension



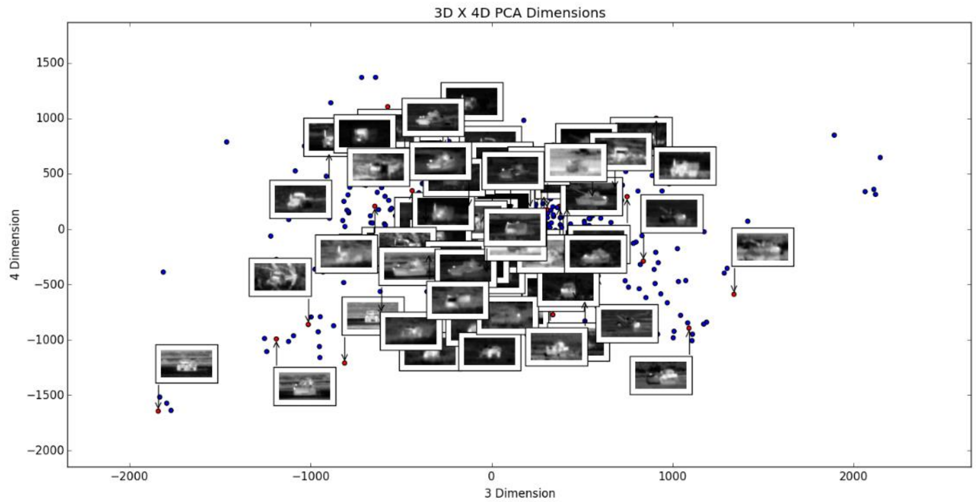
Plot 1.8: Is the plot of the second resulting of PCA dimension by the fourth resulting of PCA dimension



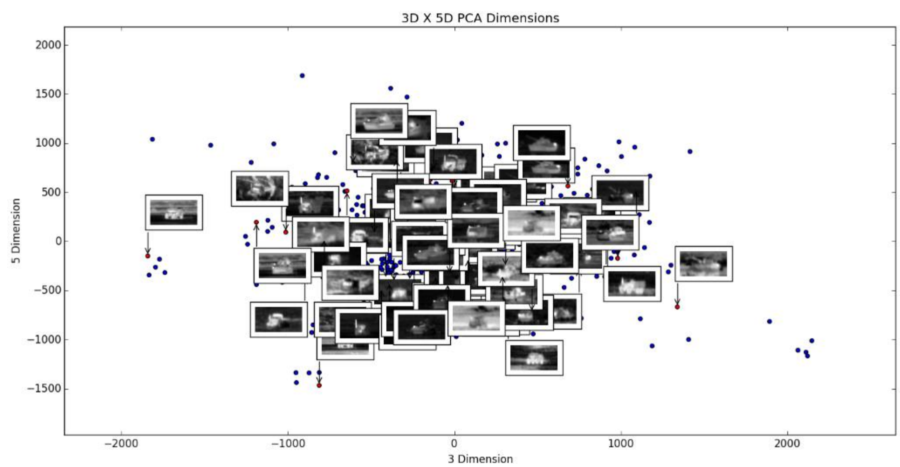
Plot 1.9: Is the plot of the second resulting of PCA dimension by the fifth resulting of PCA dimension



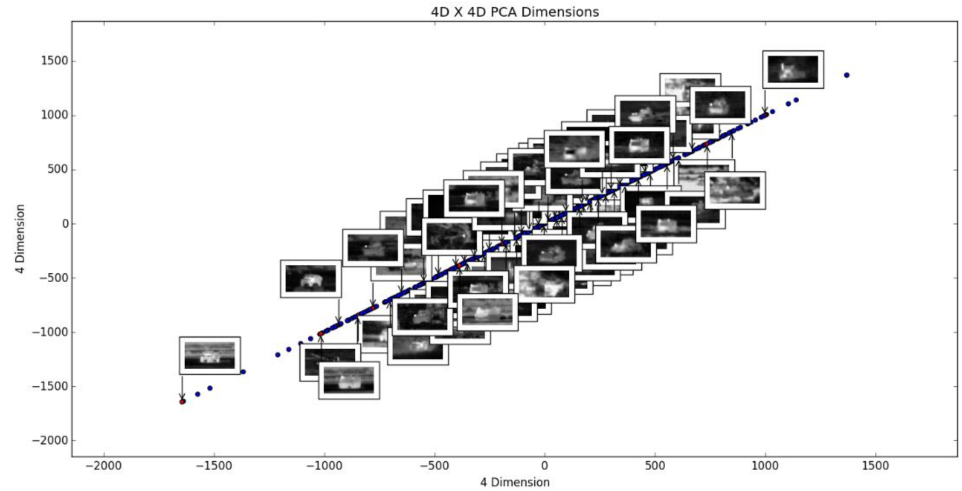
Plot 1.10: Is the plot of the third resulting of PCA dimension by the third resulting of PCA dimension



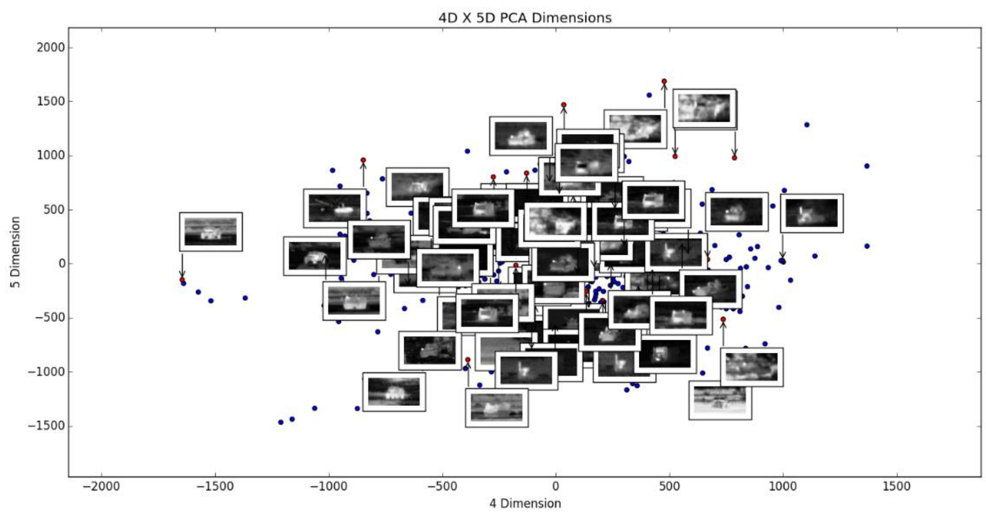
Plot 1.11: Is the plot of the third resulting of PCA dimension by the fourth resulting of PCA dimension



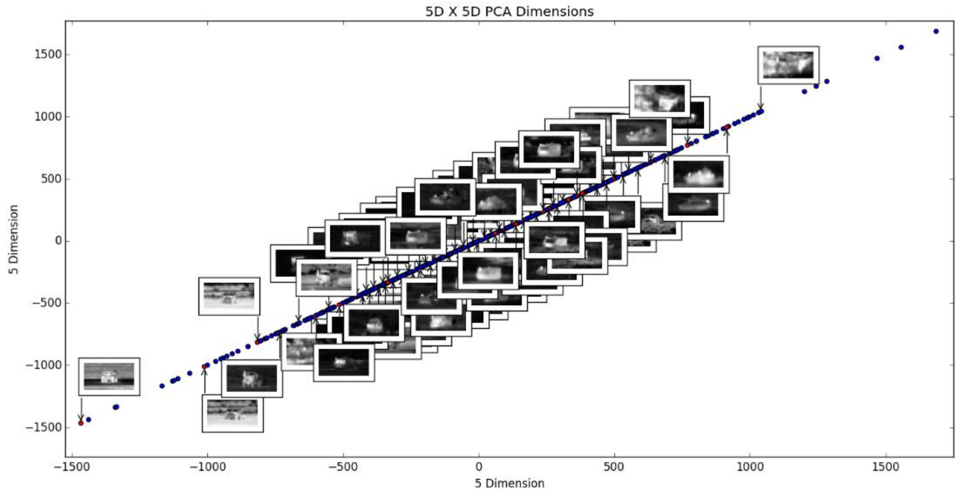
Plot 1.12: Is the plot of the third resulting of PCA dimension by the fifth resulting of PCA dimension



Plot 1.13: Is the plot of the fourth resulting of PCA dimension by the fourth resulting of PCA dimension

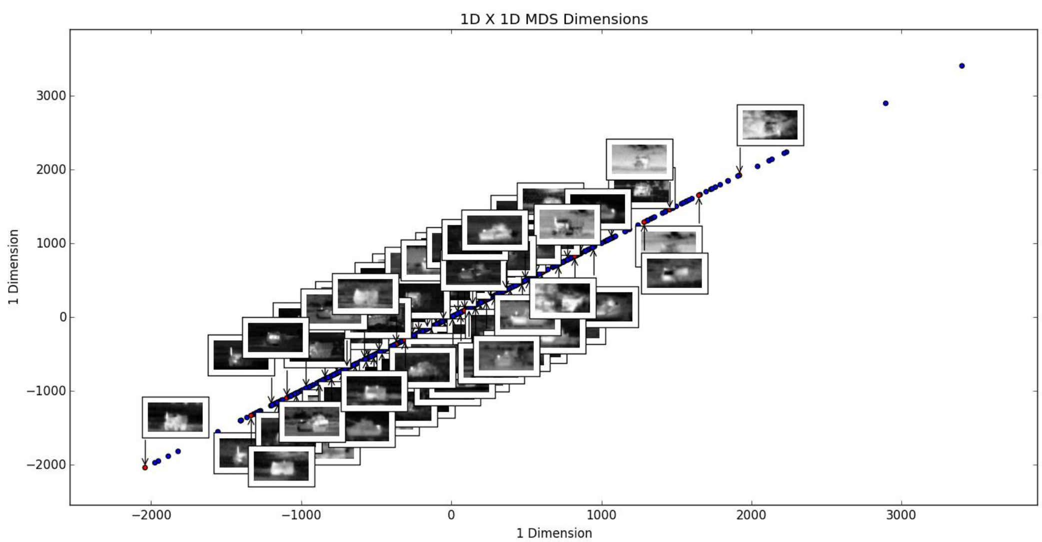


Plot 1.14: Is the plot of the fourth resulting of PCA dimension by the fifth resulting of PCA dimension

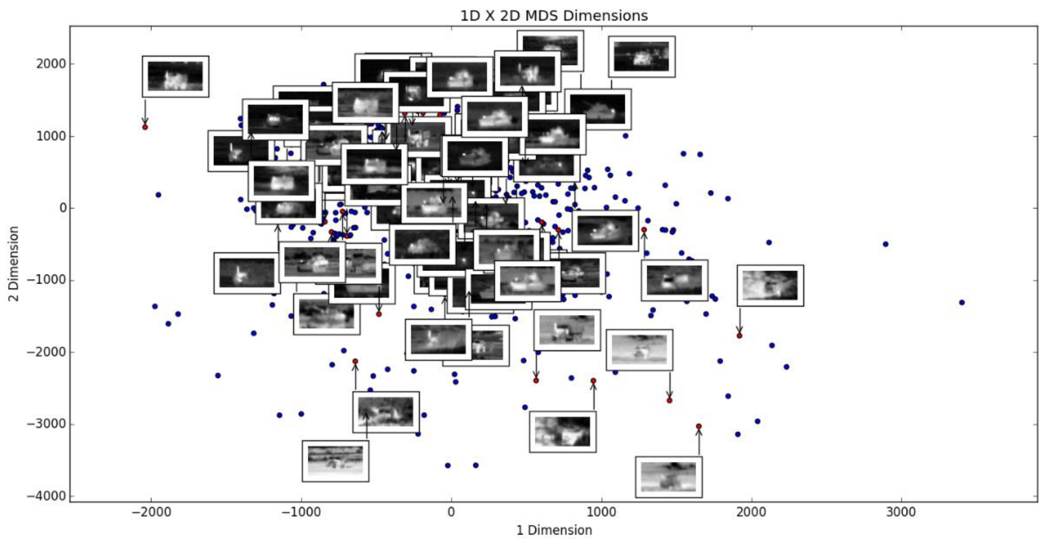


Plot 1.15: Is the plot of the fifth resulting of PCA dimension by the fifth resulting of PCA dimension

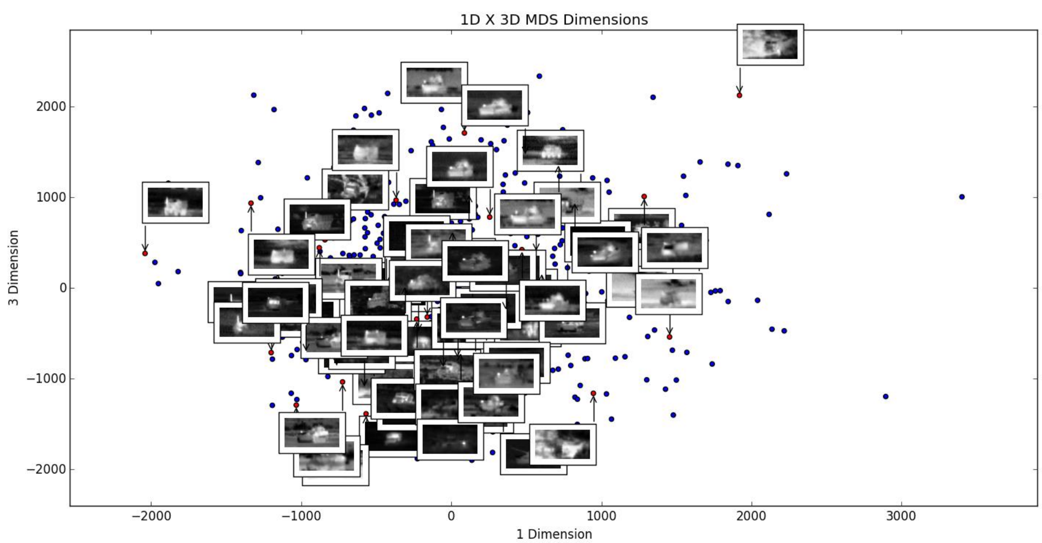
MDS



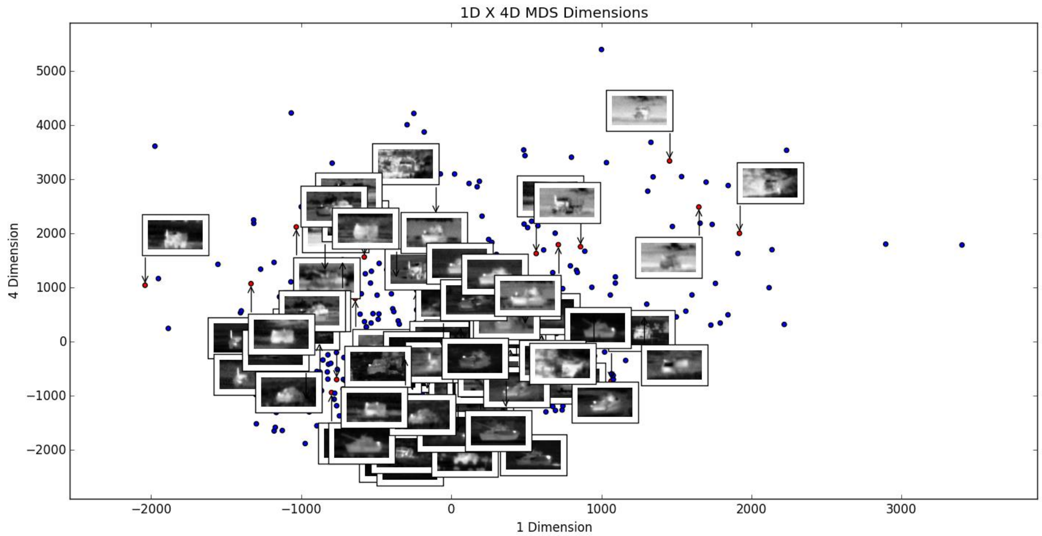
Plot 2.1: Is the plot of the first resulting of MDS dimension by the first resulting of MDS dimension



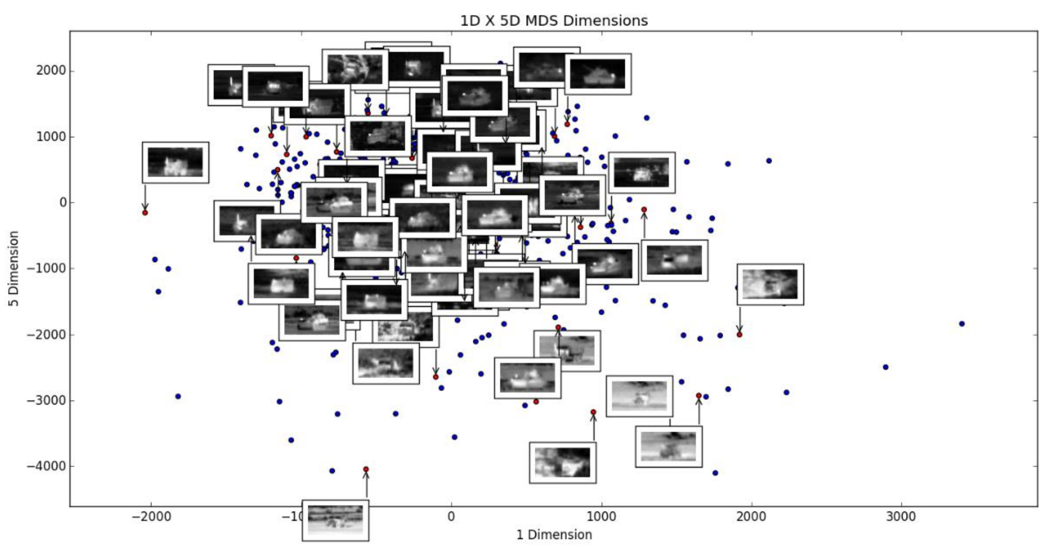
Plot 2.2: Is the plot of the first resulting of MDS dimension by the second resulting of MDS dimension



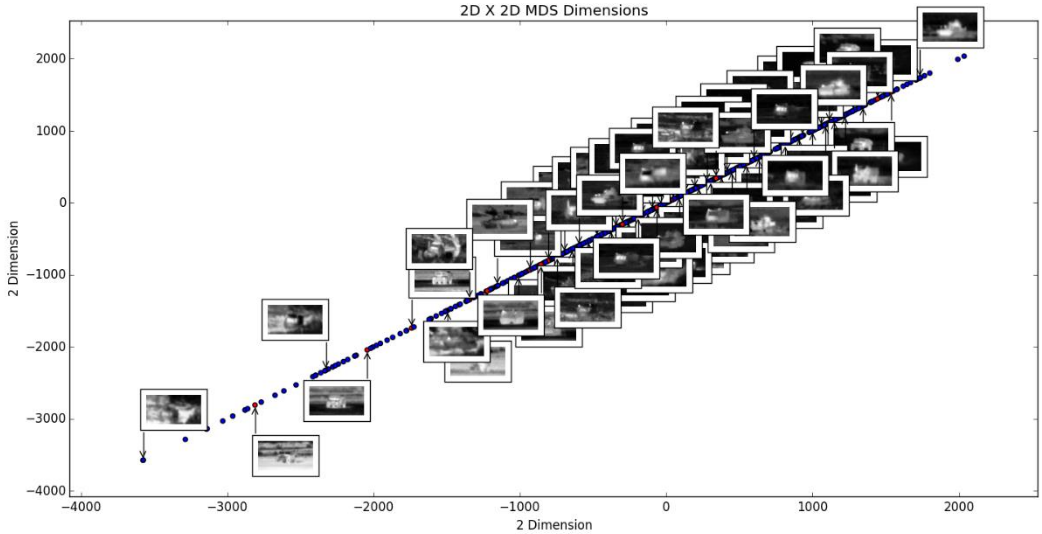
Plot 2.3: Is the plot of the first resulting of MDS dimension by the third resulting of MDS dimension



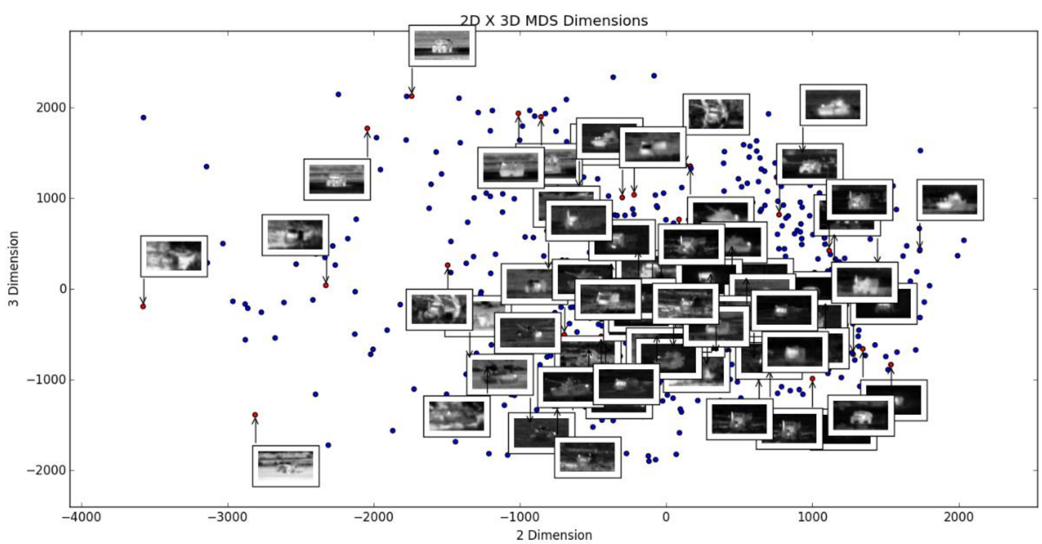
Plot 2.4: Is the plot of the first resulting of MDS dimension by the fourth resulting of MDS dimension



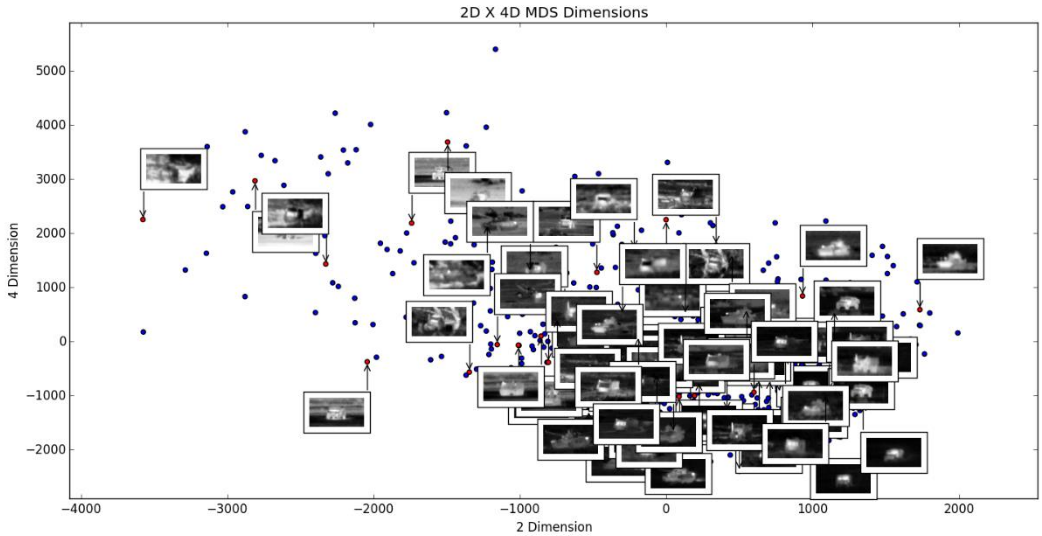
Plot 2.5: Is the plot of the first resulting of MDS dimension by the fifth resulting of MDS dimension



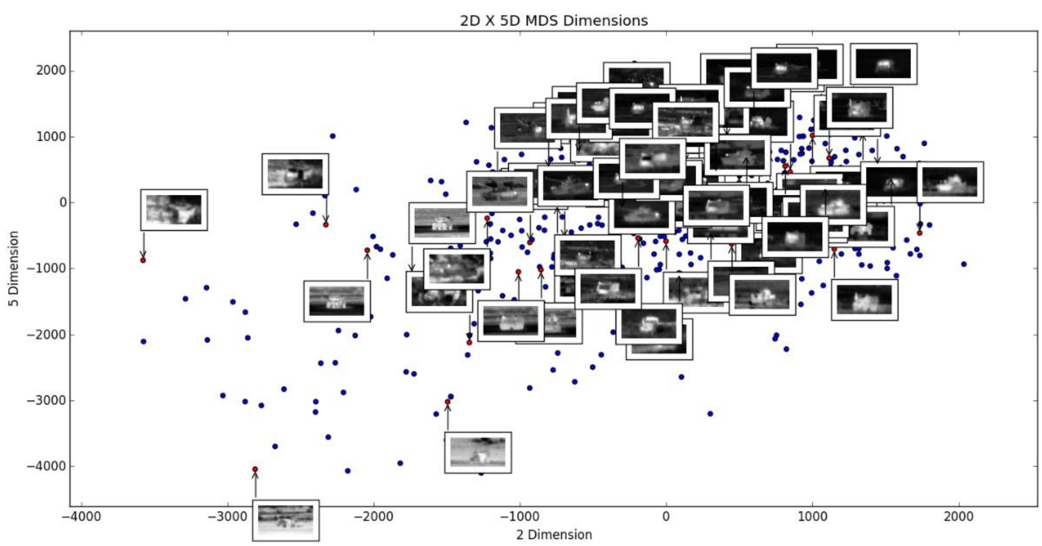
Plot 2.6: Is the plot of the second resulting of MDS dimension by the second resulting of MDS dimension



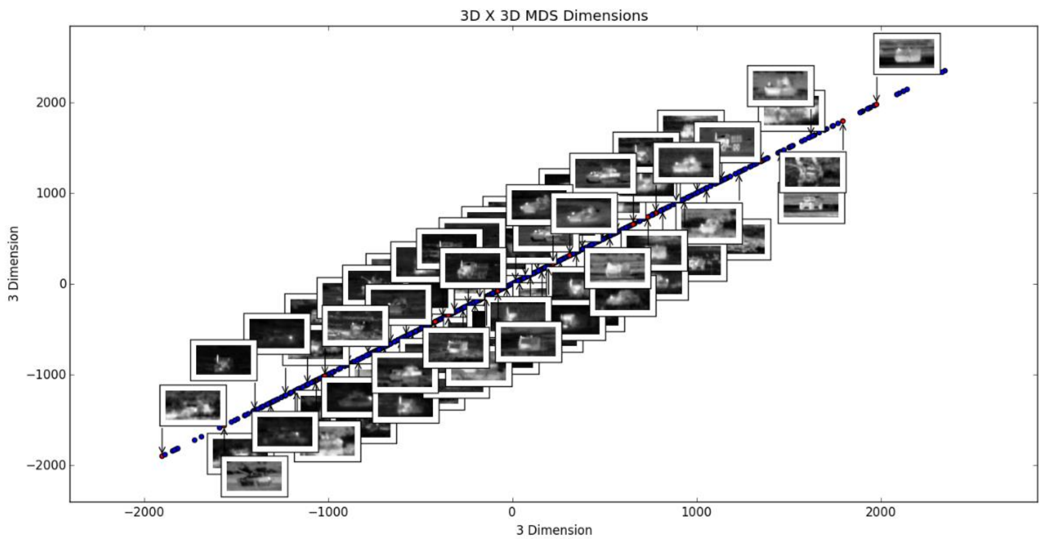
Plot 2.7: Is the plot of the second resulting of MDS dimension by the third resulting of MDS dimension



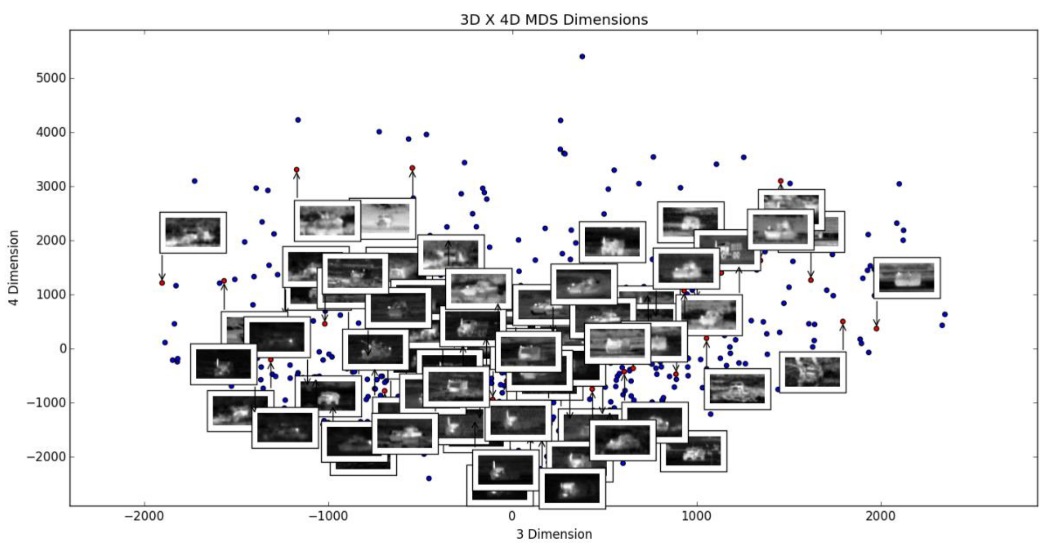
Plot 2.8: Is the plot of the second resulting of MDS dimension by the fourth resulting of MDS dimension



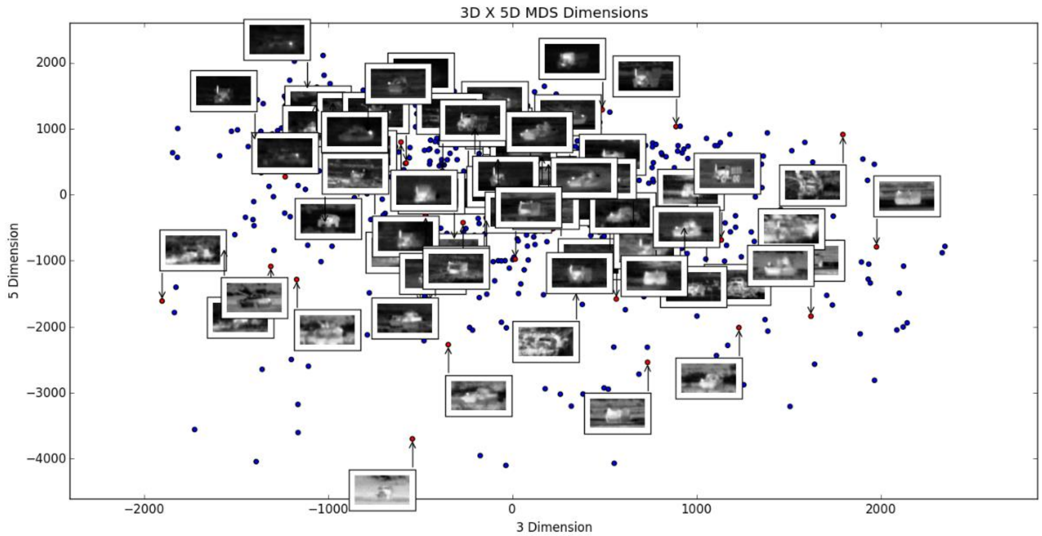
Plot 2.9: Is the plot of the second resulting of MDS dimension by the fifth resulting of MDS dimension



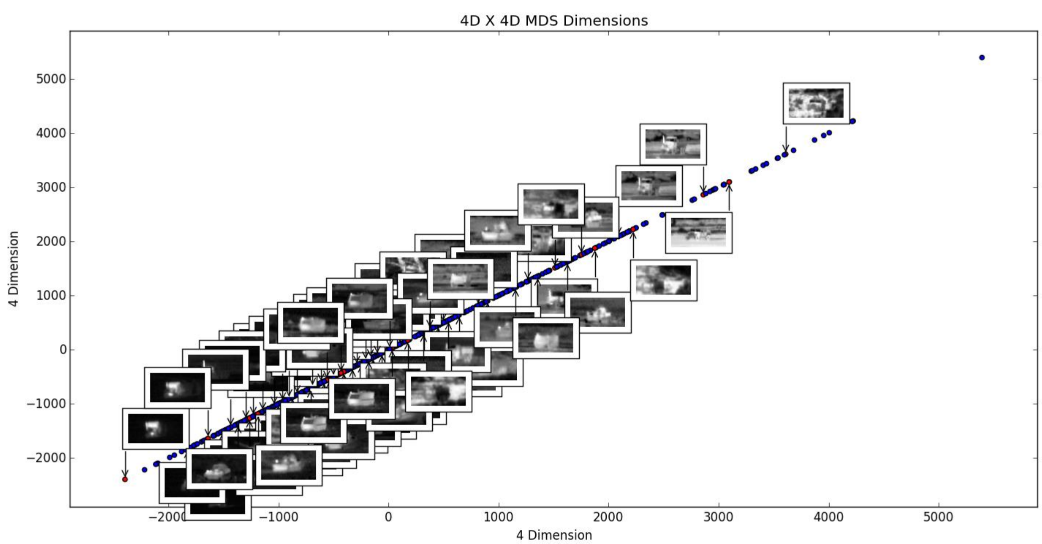
Plot 2.10: Is the plot of the third resulting of MDS dimension by the third resulting of MDS dimension



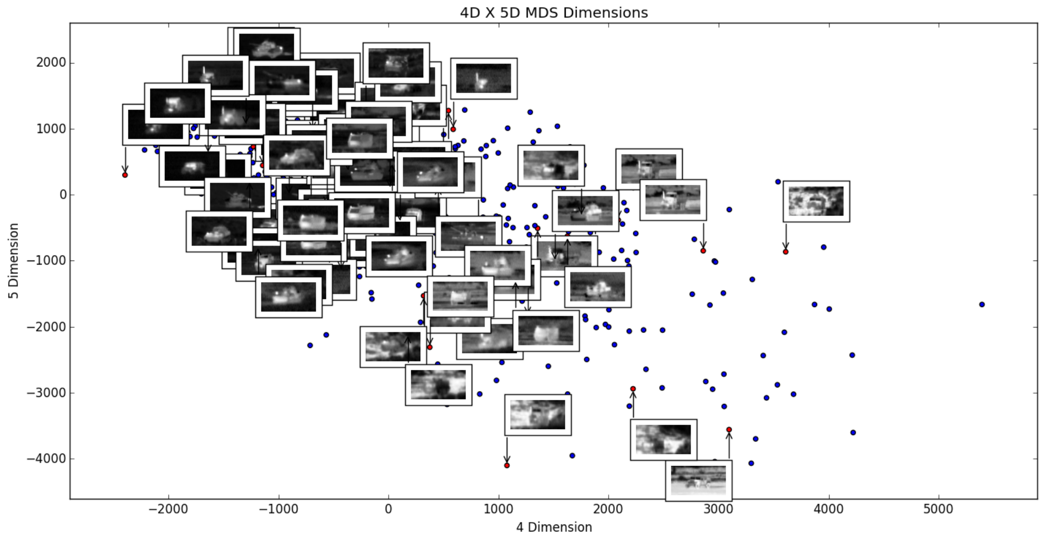
Plot 2.11: Is the plot of the third resulting of MDS dimension by the fourth resulting of MDS dimension



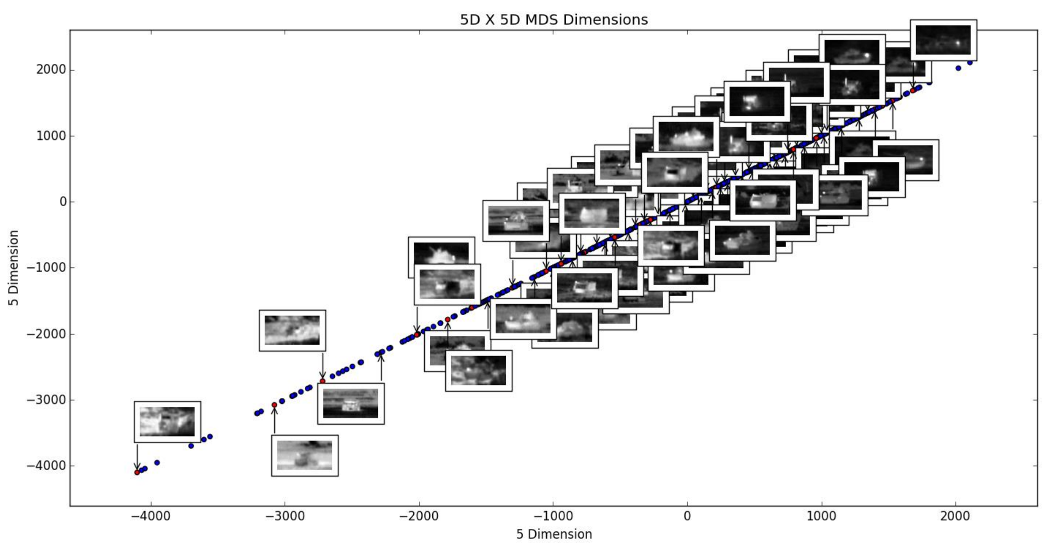
Plot 2.12: Is the plot of the third resulting of MDS dimension by the fifth resulting of MDS dimension



Plot 2.13: Is the plot of the fourth resulting of MDS dimension by the fourth resulting of MDS dimension

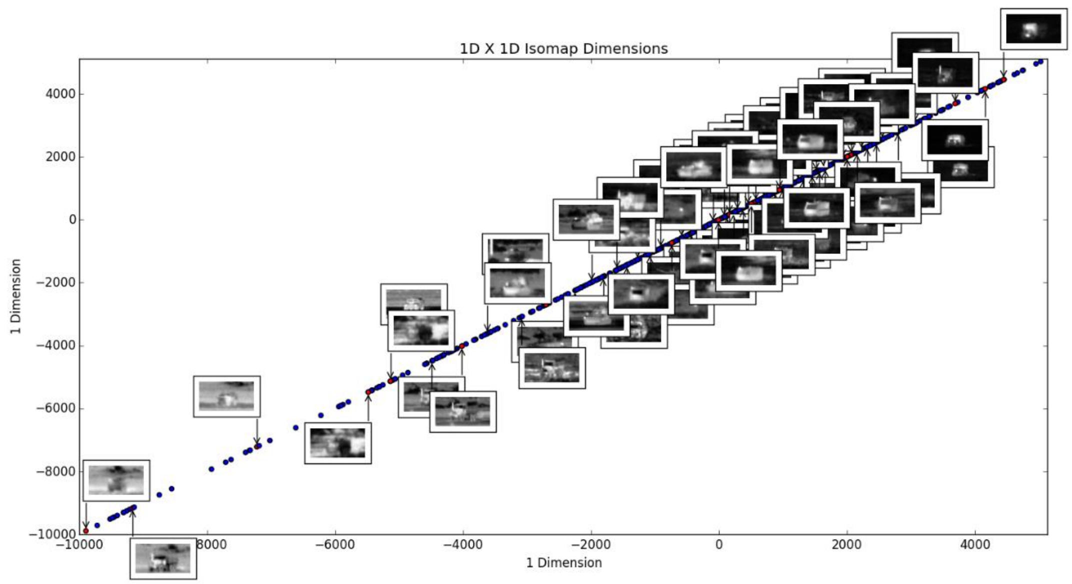


Plot 2.14: Is the plot of the fourth resulting of MDS dimension by the fifth resulting of MDS dimension

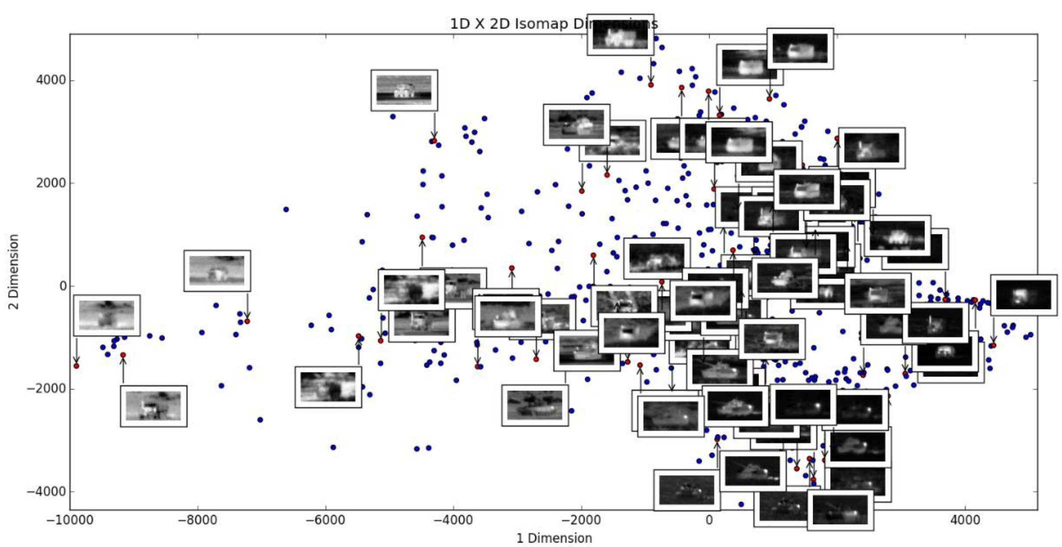


Plot 2.15: Is the plot of the fifth resulting of MDS dimension by the fifth resulting of MDS dimension

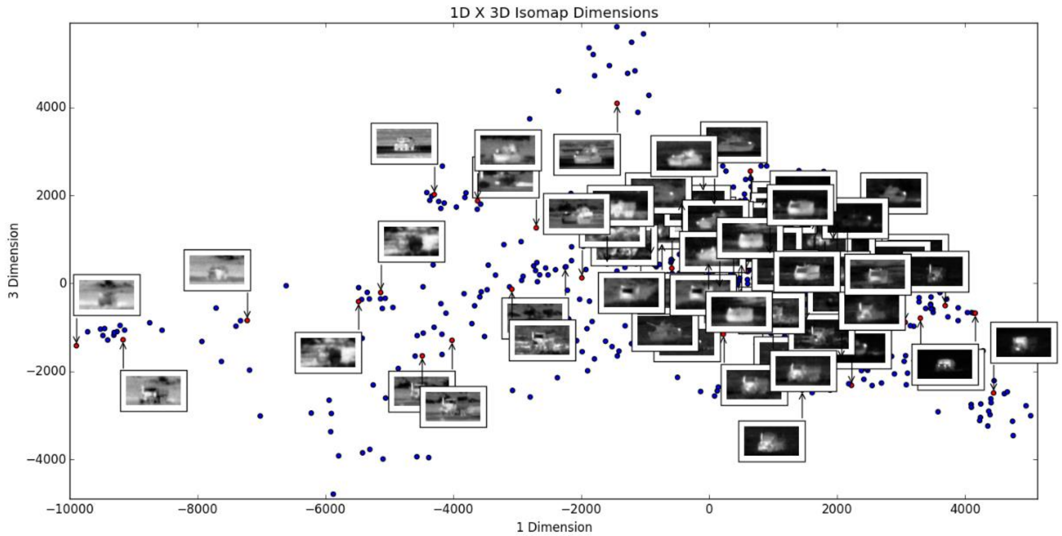
Isomap



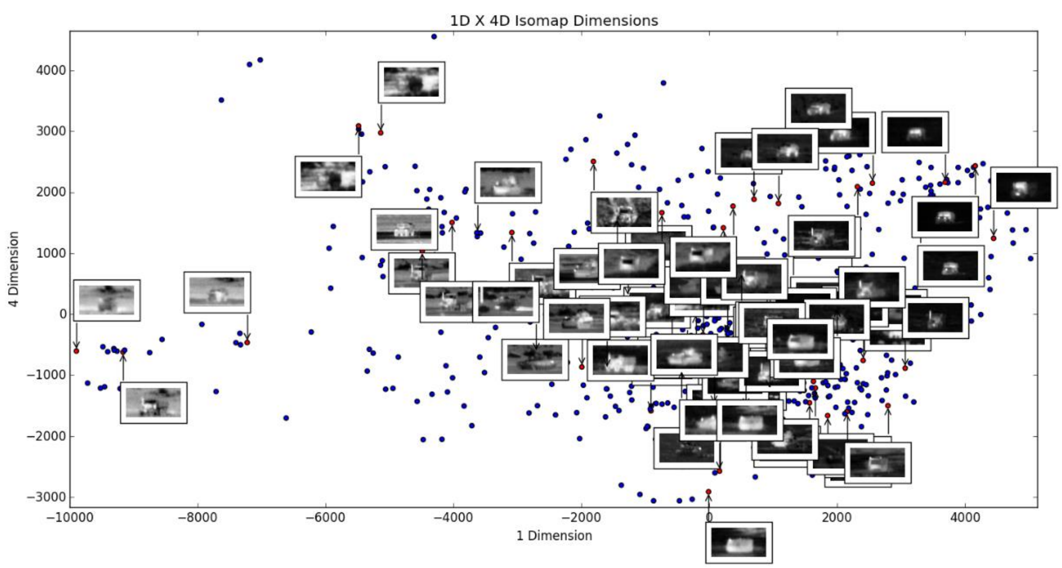
Plot 3.1: Is the plot of the first resulting of Isomap dimension by the first resulting of Isomap dimension



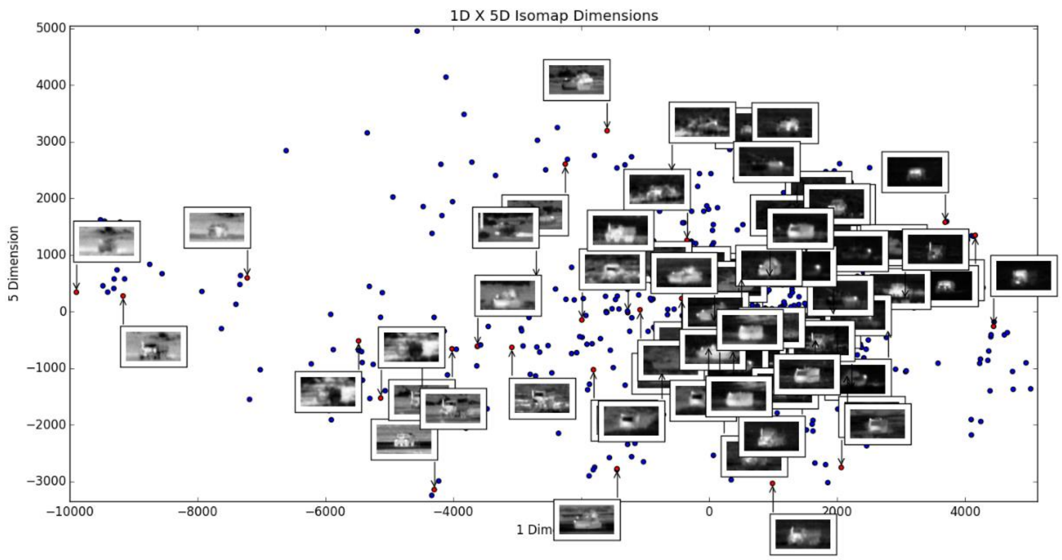
Plot 3.2: Is the plot of the first resulting of Isomap dimension by the second resulting of Isomap dimension



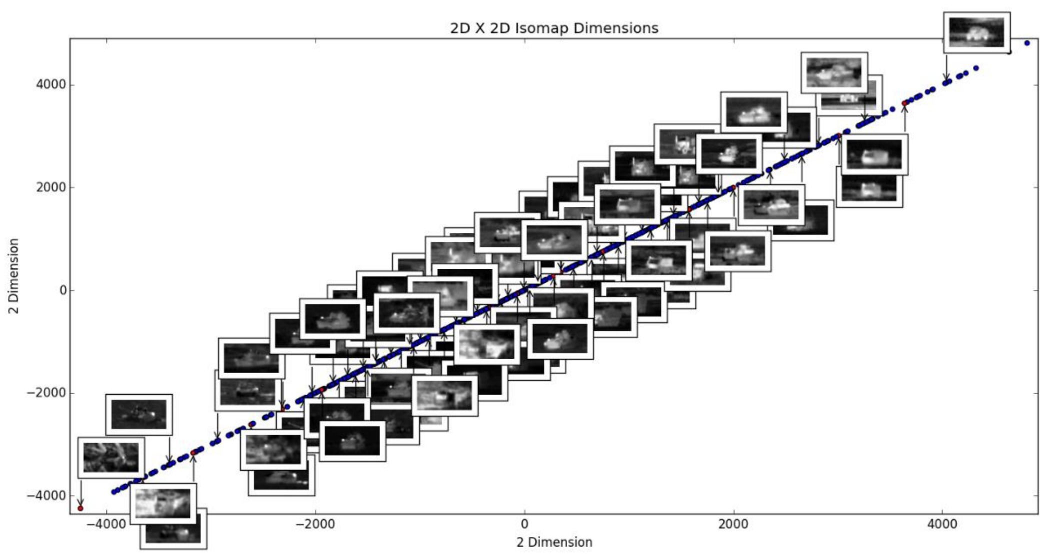
Plot 3.3: Is the plot of the first resulting of Isomap dimension by the third resulting of Isomap dimension



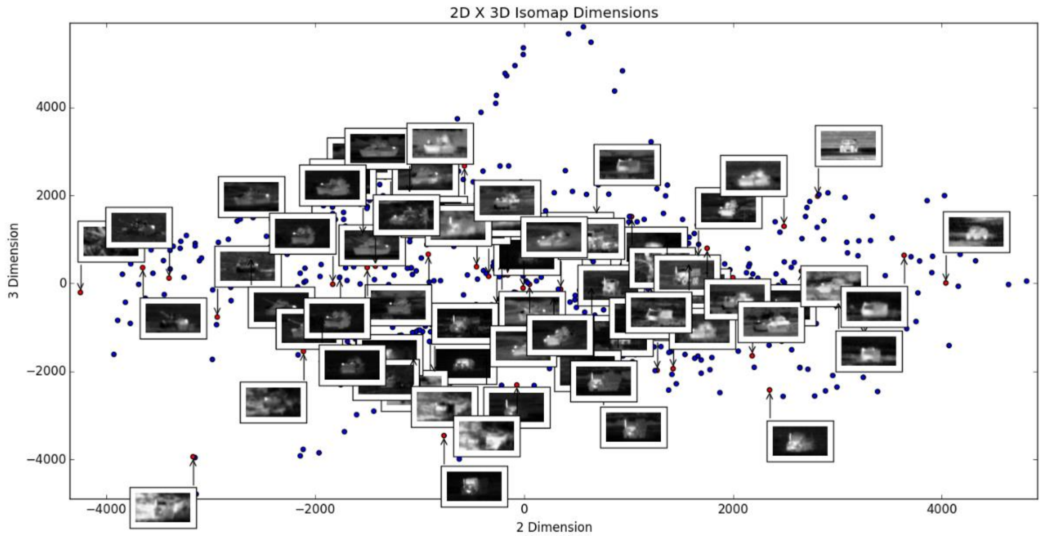
Plot 3.4: Is the plot of the first resulting of Isomap dimension by the fourth resulting of Isomap dimension



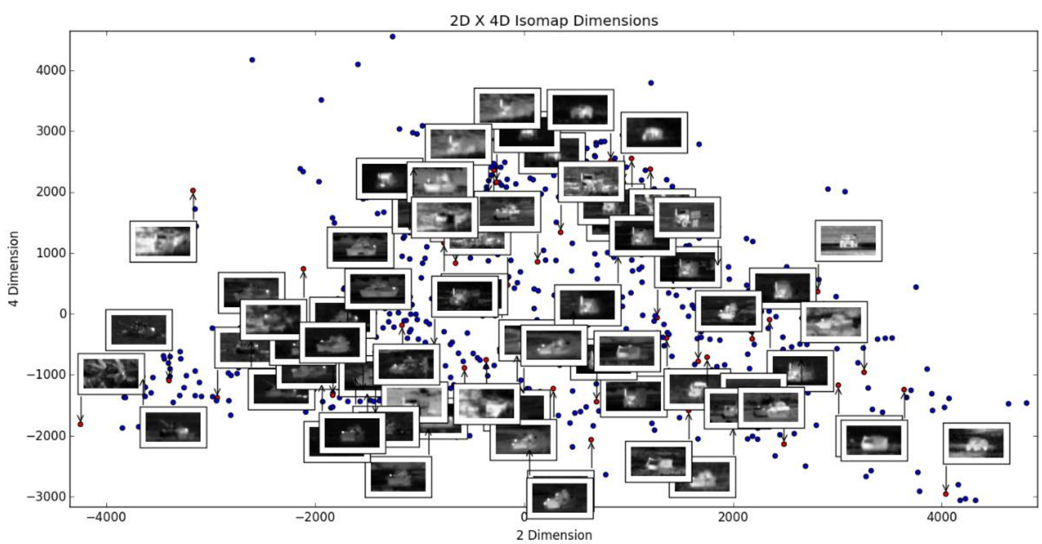
Plot 3.5: Is the plot of the first resulting of Isomap dimension by the fifth resulting of Isomap dimension



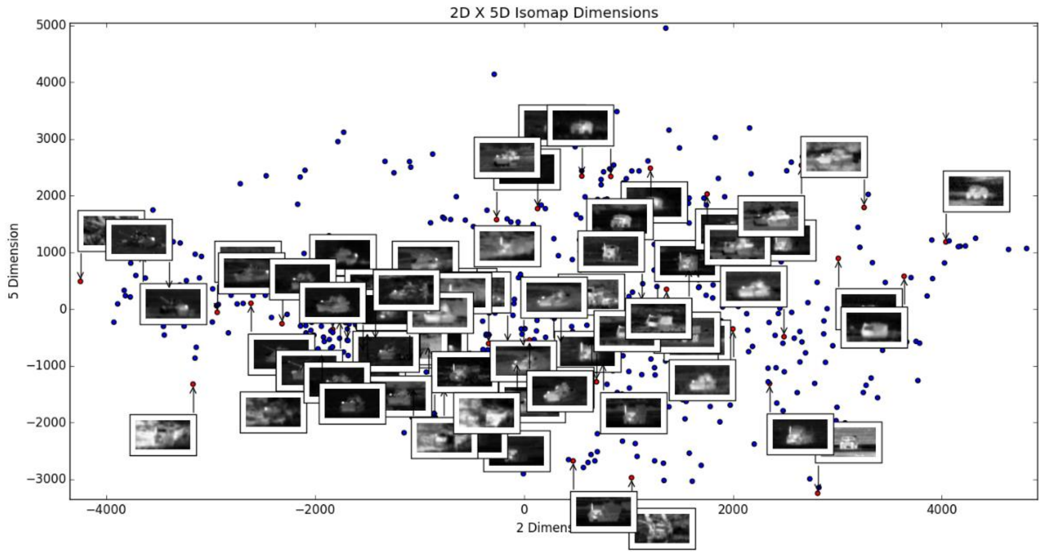
Plot 3.6: Is the plot of the second resulting of Isomap dimension by the second resulting of Isomap dimension



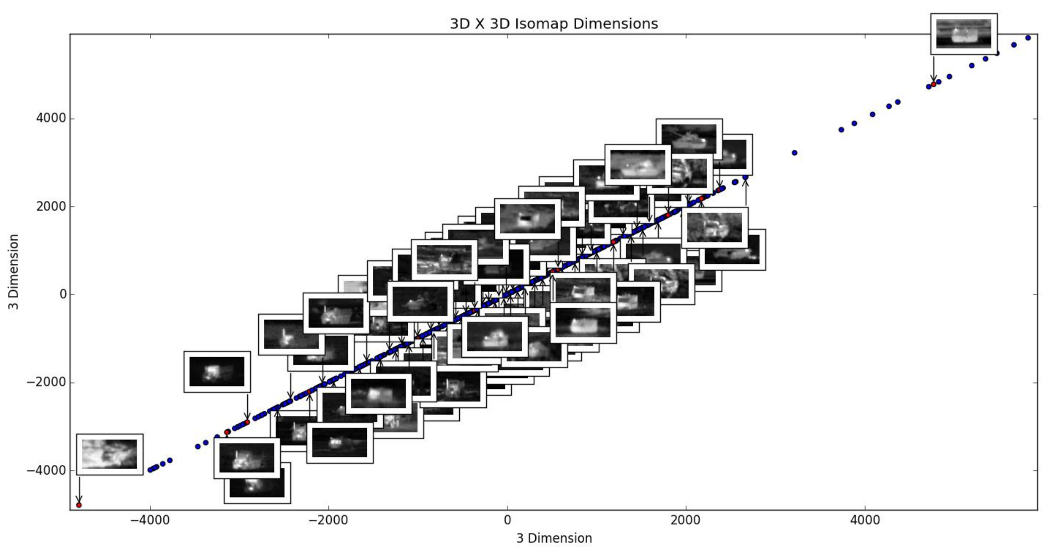
Plot 3.7: Is the plot of the second resulting of Isomap dimension by the third resulting of Isomap dimension



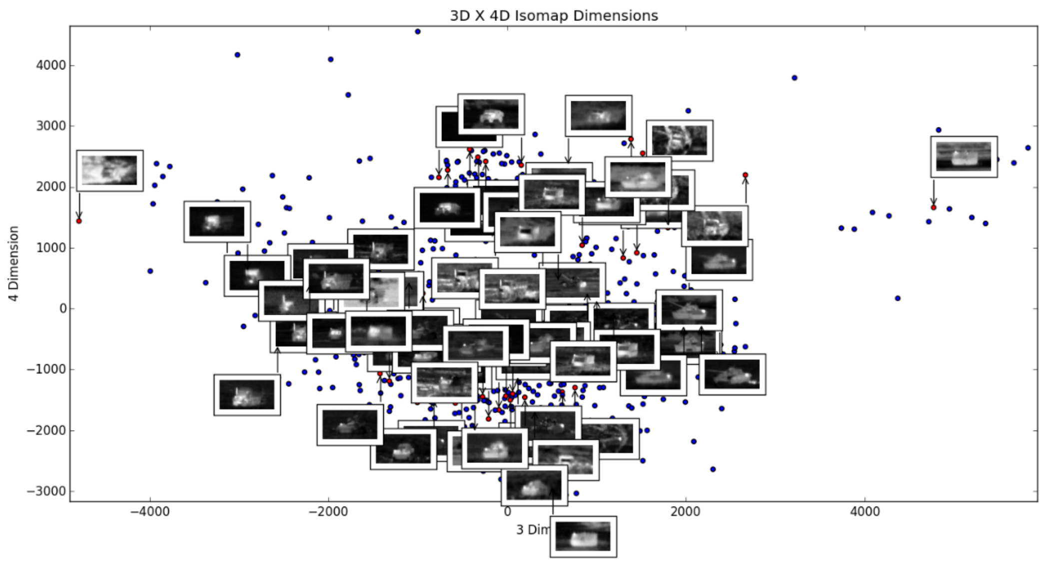
Plot 3.8: Is the plot of the second resulting of Isomap dimension by the fourth resulting of Isomap dimension



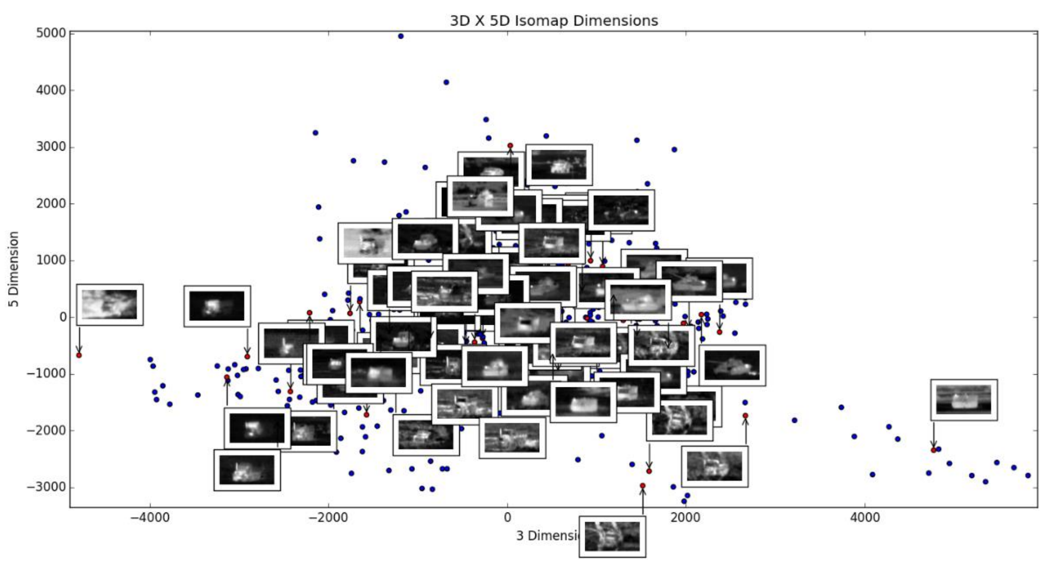
Plot 3.9: Is the plot of the second resulting of Isomap dimension by the fifth resulting of Isomap dimension



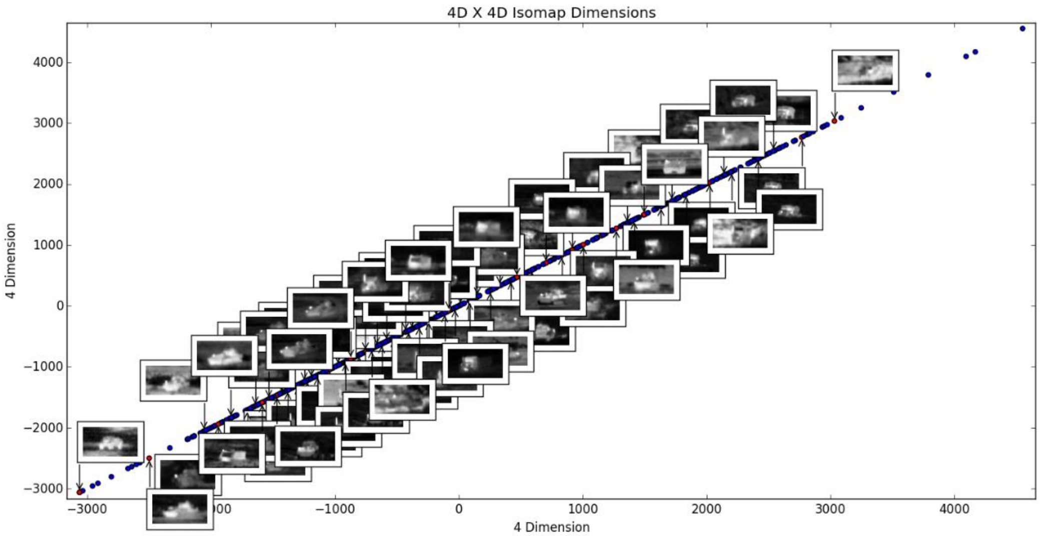
Plot 3.10: Is the plot of the third resulting of Isomap dimension by the third resulting of Isomap dimension



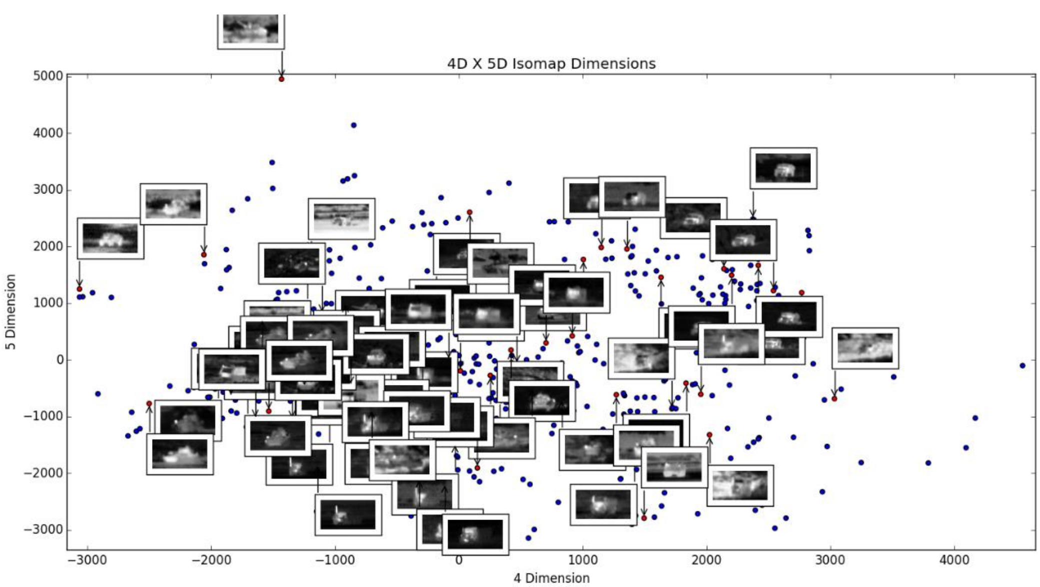
Plot 3.11: Is the plot of the third resulting of Isomap dimension by the fourth resulting of Isomap dimension



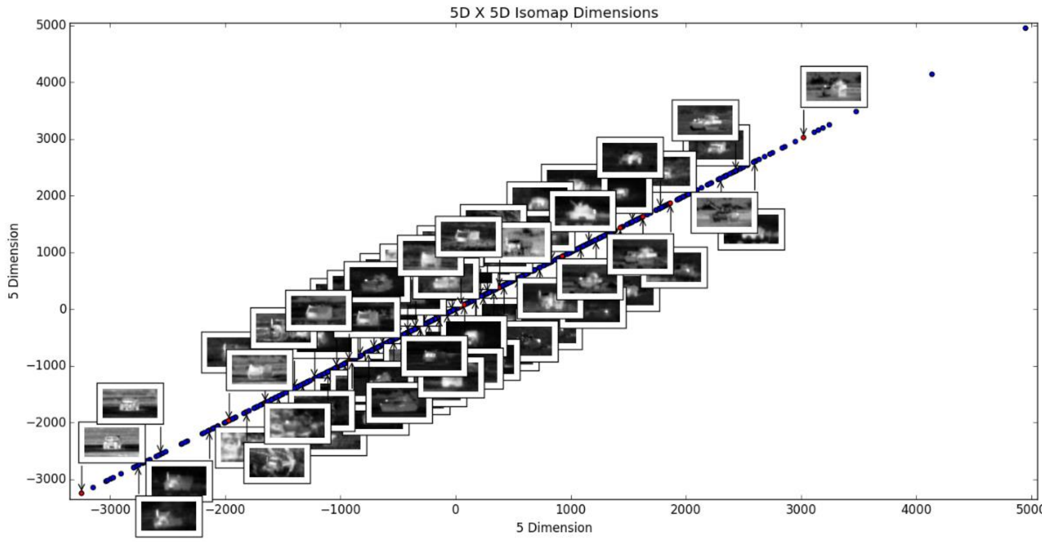
Plot 3.12: Is the plot of the third resulting of Isomap dimension by the fifth resulting of Isomap dimension



Plot 3.13: Is the plot of the fourth resulting of Isomap dimension by the fourth resulting of Isomap dimension

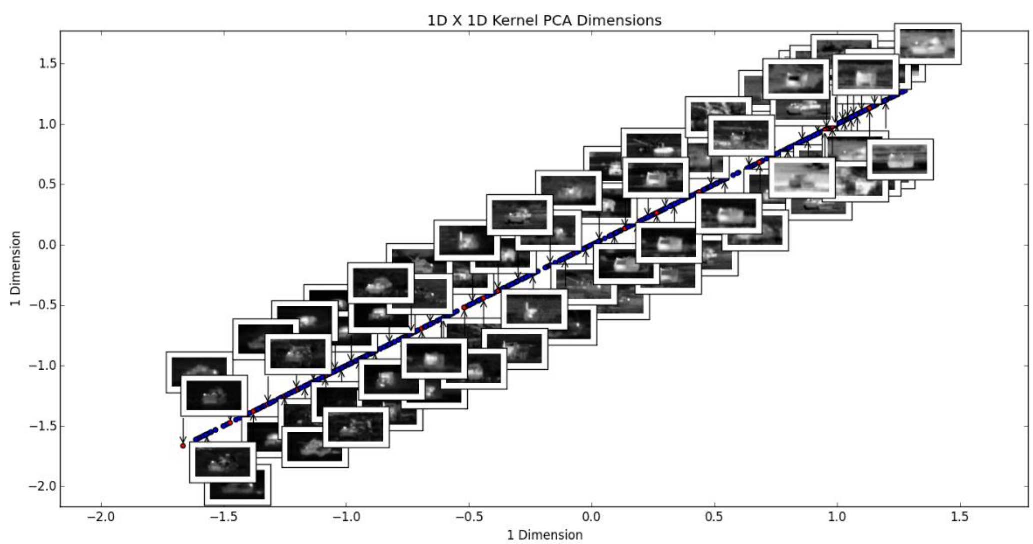


Plot 3.14: Is the plot of the fourth resulting of Isomap dimension by the fifth resulting of Isomap dimension

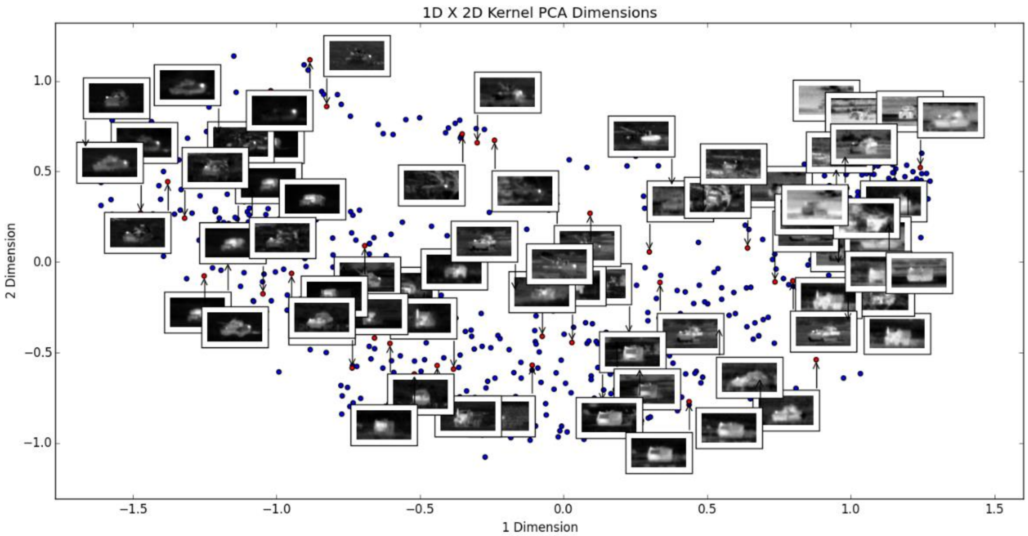


Plot 3.15: Is the plot of the fifth resulting of Isomap dimension by the fifth resulting of Isomap dimension

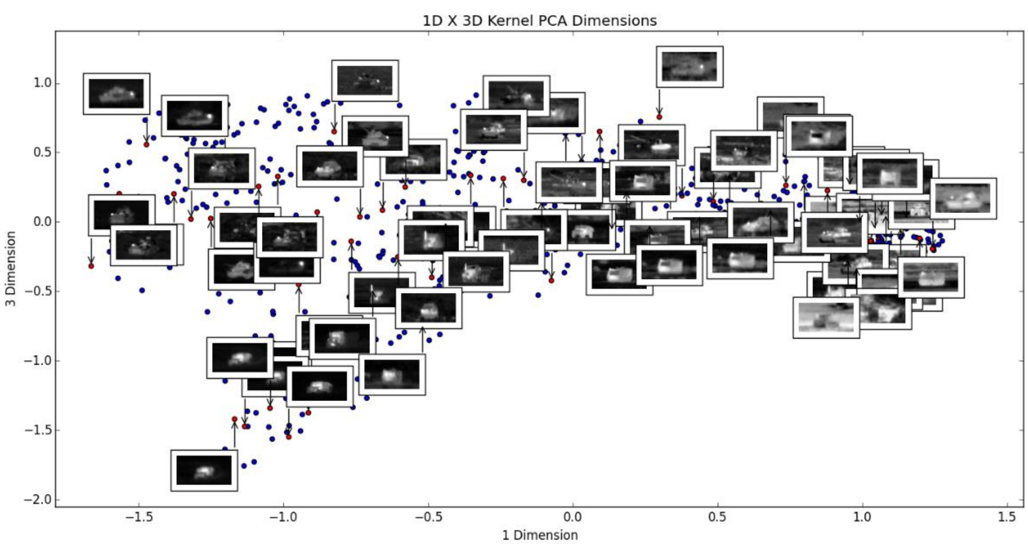
Kernel PCA



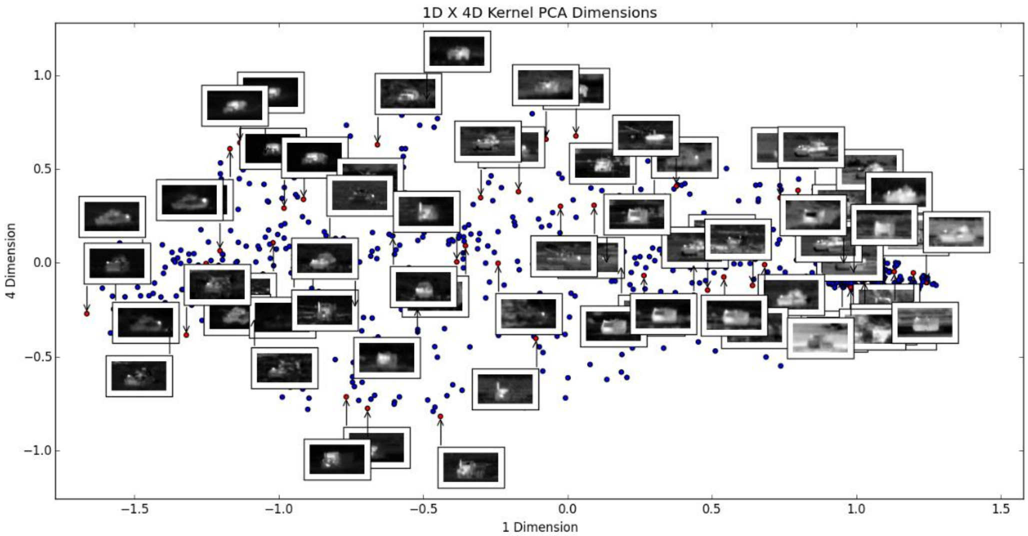
Plot 4.1: Is the plot of the first resulting of Kernel PCA dimension by the first resulting of Kernel PCA dimension



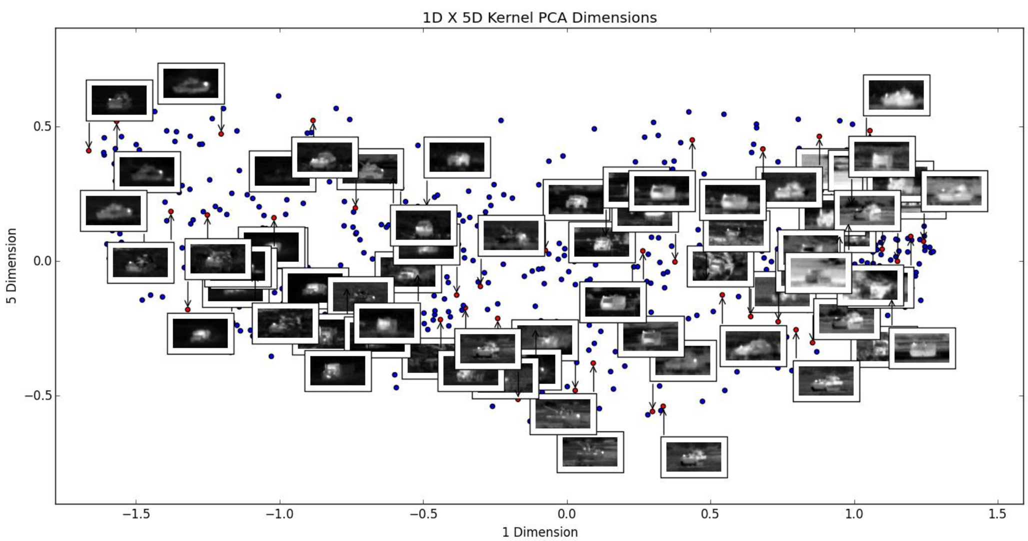
Plot 4.2: Is the plot of the first resulting of Kernel PCA dimension by the second resulting of Kernel PCA dimension



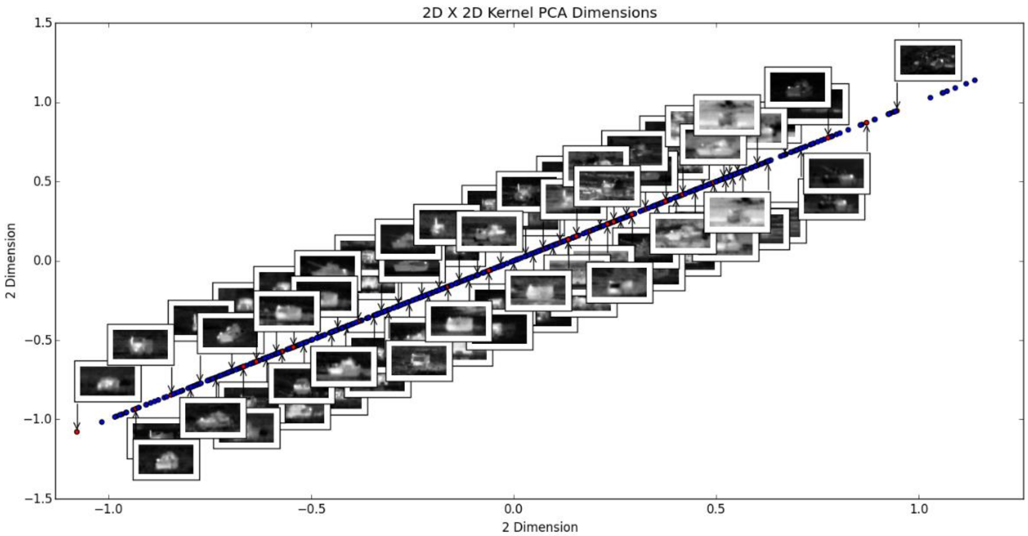
Plot 4.3: Is the plot of the first resulting of Kernel PCA dimension by the third resulting of Kernel PCA dimension



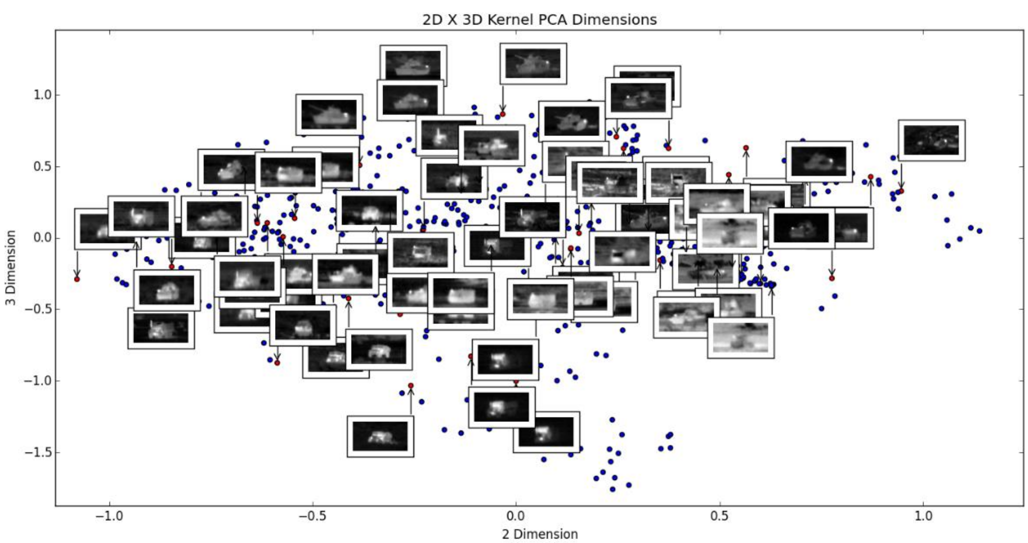
Plot 4.4: Is the plot of the first resulting of Kernel PCA dimension by the fourth resulting of Kernel PCA dimension



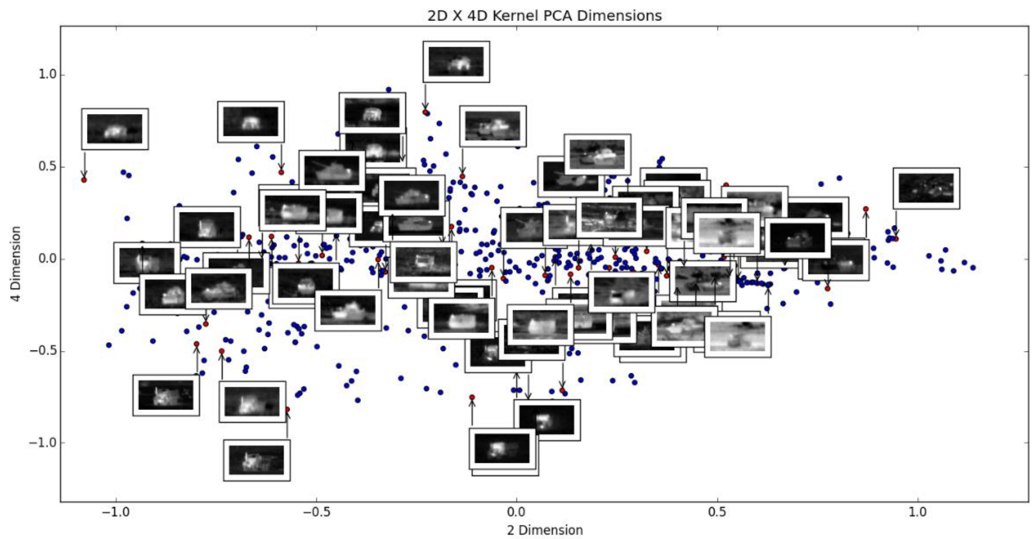
Plot 4.5: Is the plot of the first resulting of Kernel PCA dimension by the fifth resulting of Kernel PCA dimension



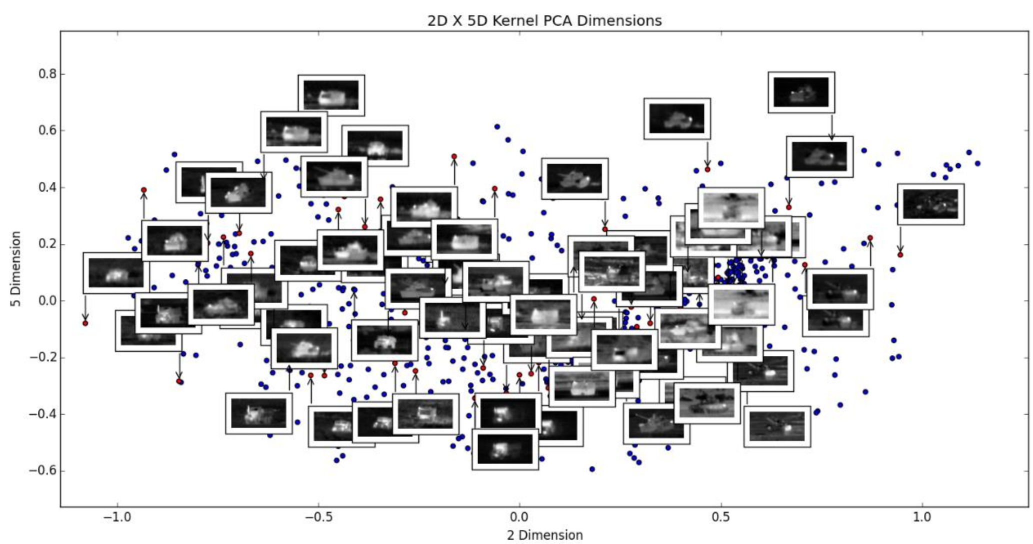
Plot 4.6: Is the plot of the second resulting of Kernel PCA dimension by the second resulting of Kernel PCA dimension



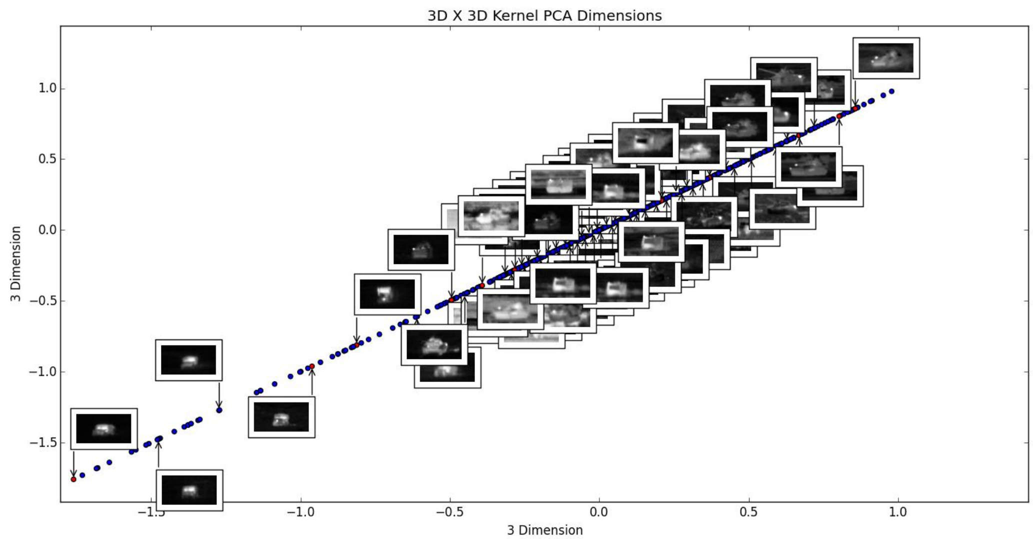
Plot 4.7: Is the plot of the second resulting of Kernel PCA dimension by the third resulting of Kernel PCA dimension



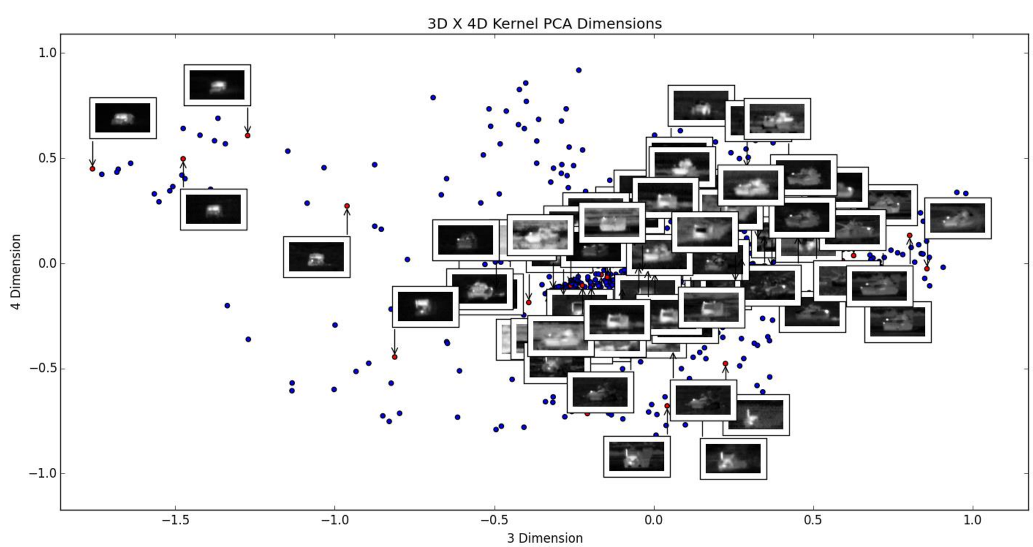
Plot 4.8: Is the plot of the second resulting of Kernel PCA dimension by the fourth resulting of Kernel PCA dimension



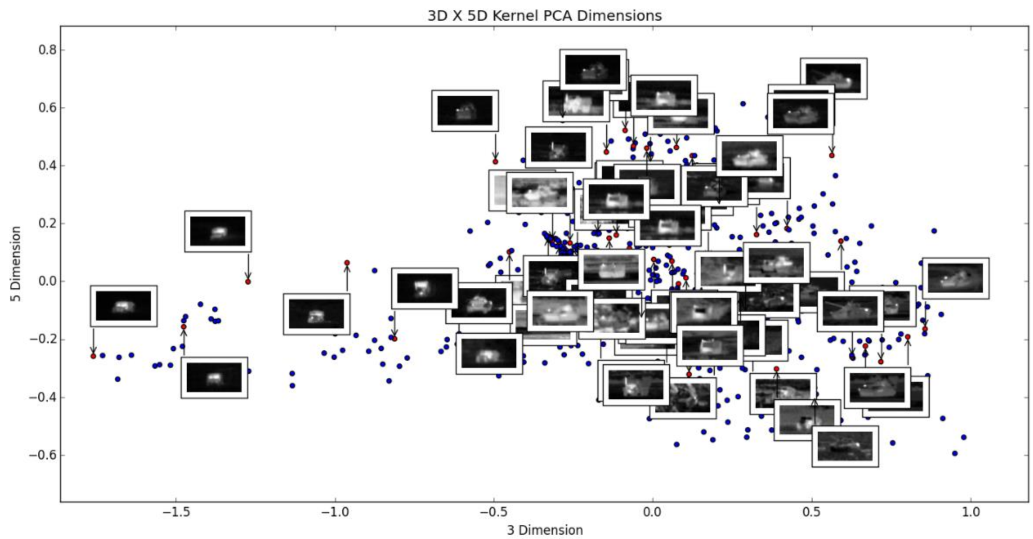
Plot 4.9: Is the plot of the second resulting of Kernel PCA dimension by the fifth resulting of Kernel PCA dimension



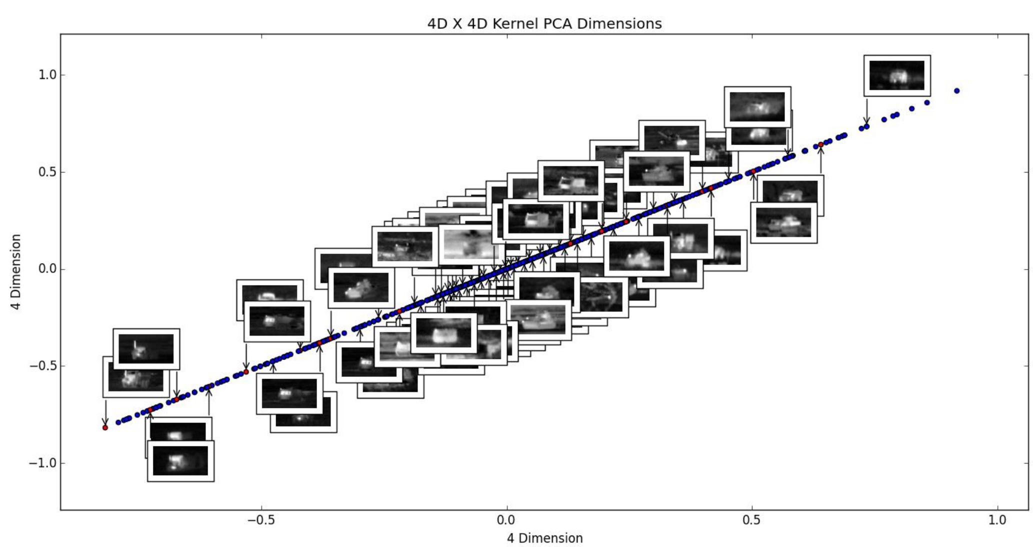
Plot 4.10: Is the plot of the third resulting of Kernel PCA dimension by the third resulting of Kernel PCA dimension



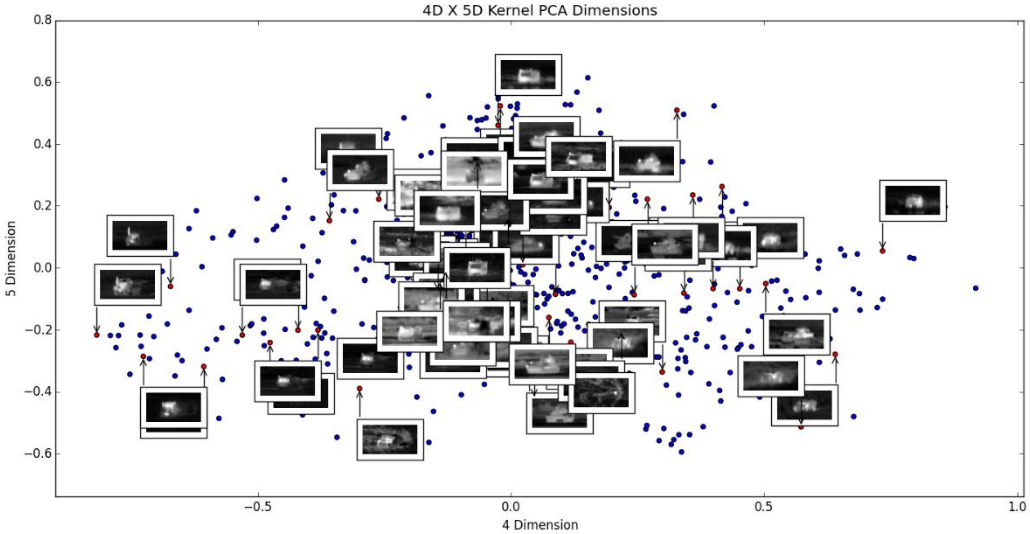
Plot 4.11: Is the plot of the third resulting of Kernel PCA dimension by the fourth resulting of Kernel PCA dimension



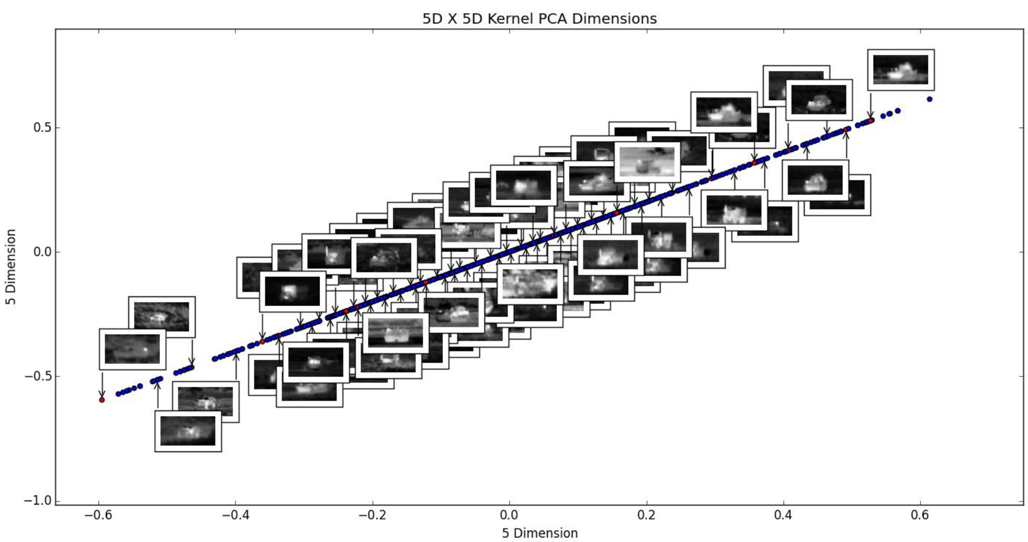
Plot 4.12: Is the plot of the third resulting of Kernel PCA dimension by the third resulting of Kernel PCA dimension



Plot 4.13: Is the plot of the fourth resulting of Kernel PCA dimension by the fourth resulting of Kernel PCA dimension



Plot 4.14: Is the plot of the fourth resulting of Kernel PCA dimension by the fifth resulting of Kernel PCA dimension



Plot 4.15: Is the plot of the fifth resulting of Kernel PCA dimension by the fifth resulting of Kernel PCA dimension

Linear Imaging Results

PCA

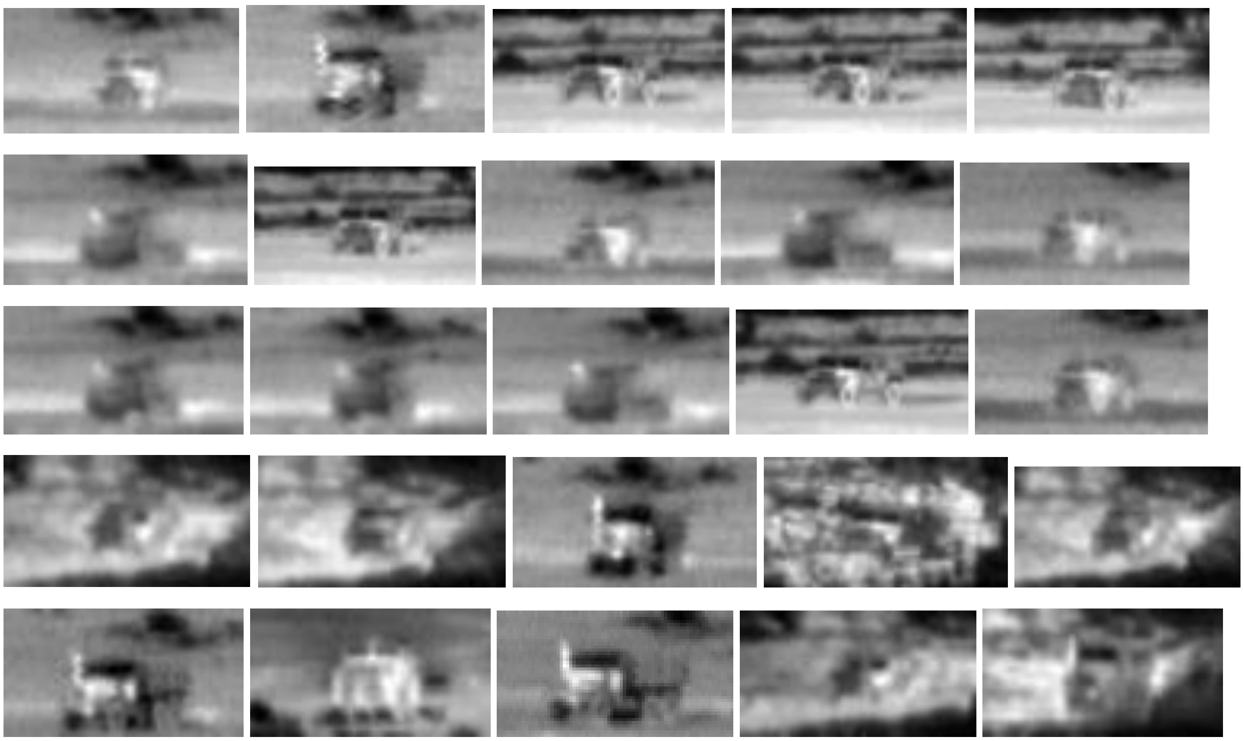
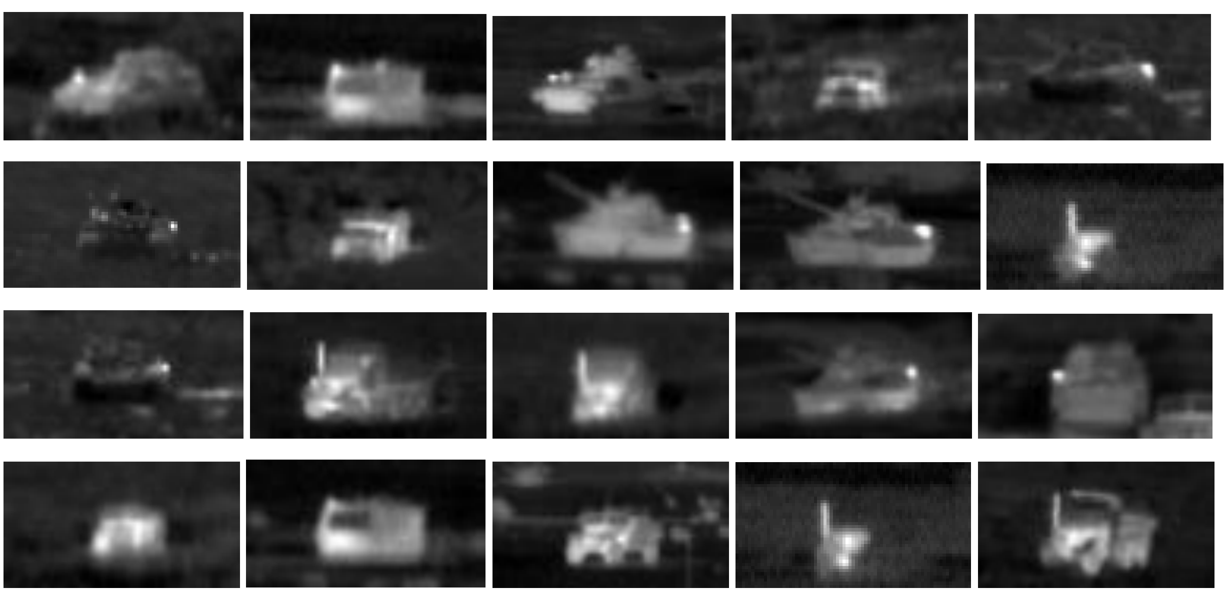


Figure 5.1.1. First 25 images of the first dimension of PCA



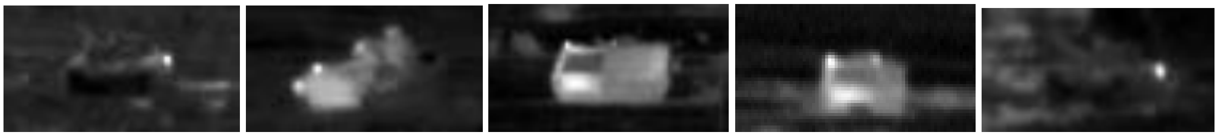


Figure 5.1.2. Middle 25 images of the first dimension of PCA

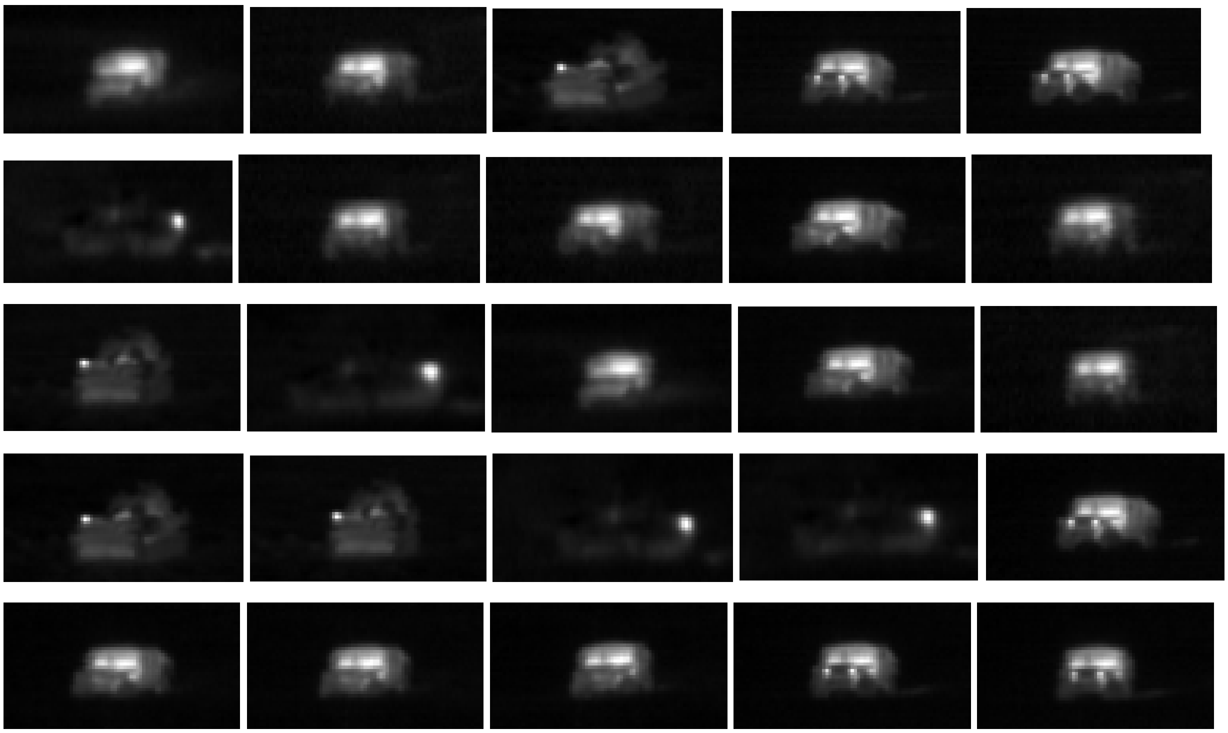
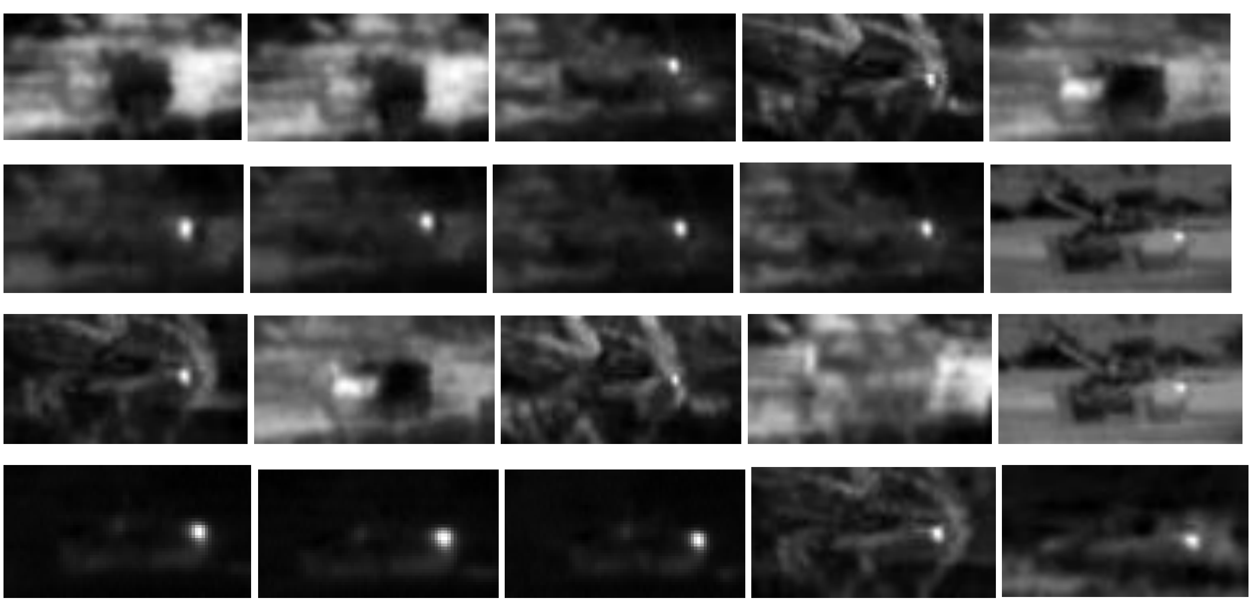


Figure 5.1.3. Last 25 images of the first dimension of PCA



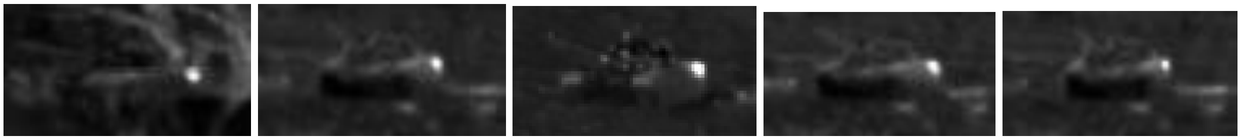


Figure 5.2.1. First 25 images of the second dimension of PCA

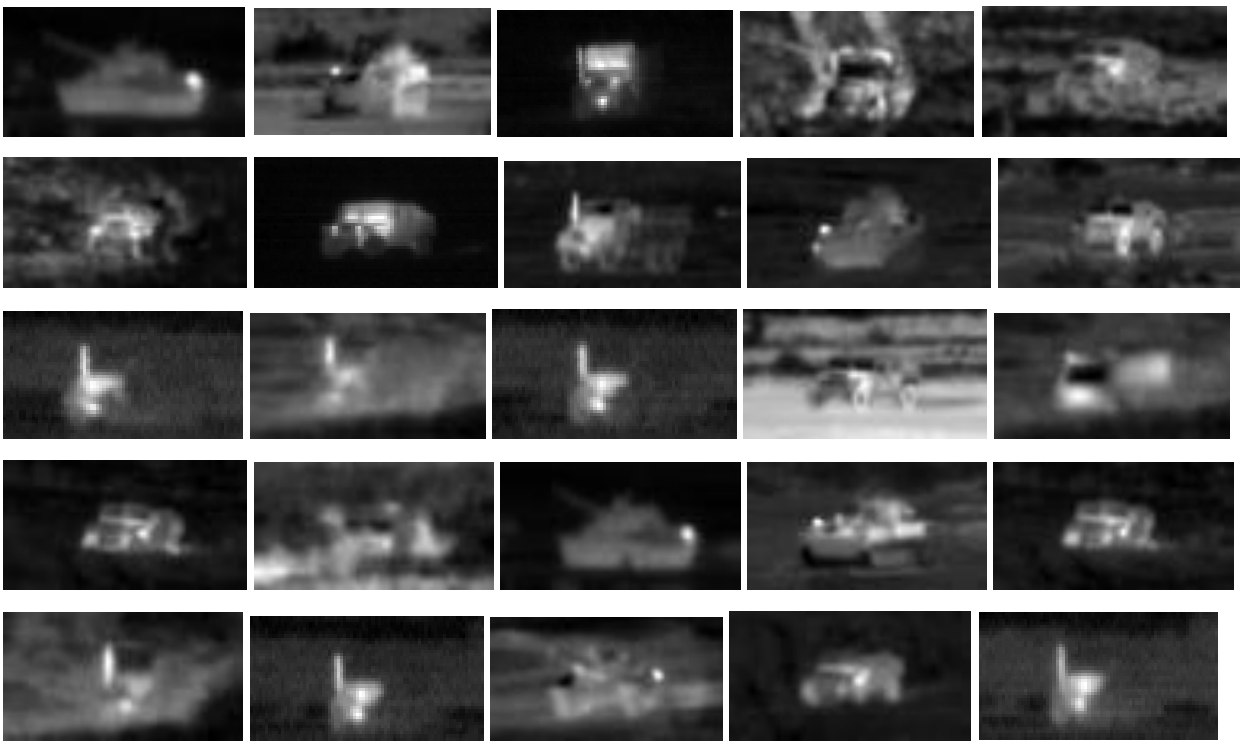
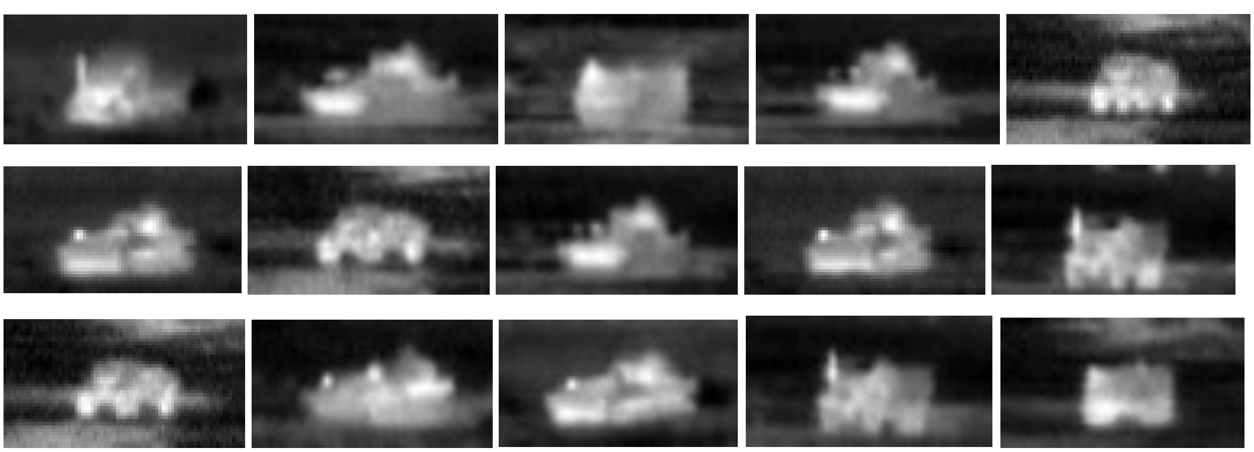


Figure 5.2.2. Middle 25 images of the second dimension of PCA



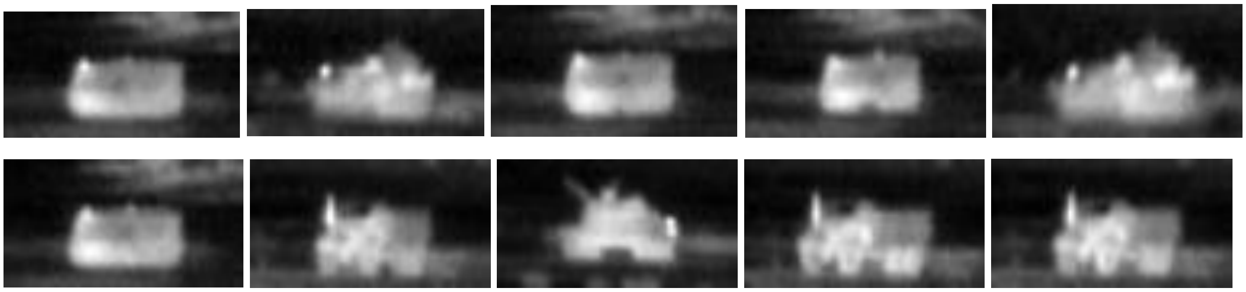


Figure 5.2.3. Last 25 images of the second dimension of PCA

Figure 5.3.1. First 25 images of the third dimension of PCA

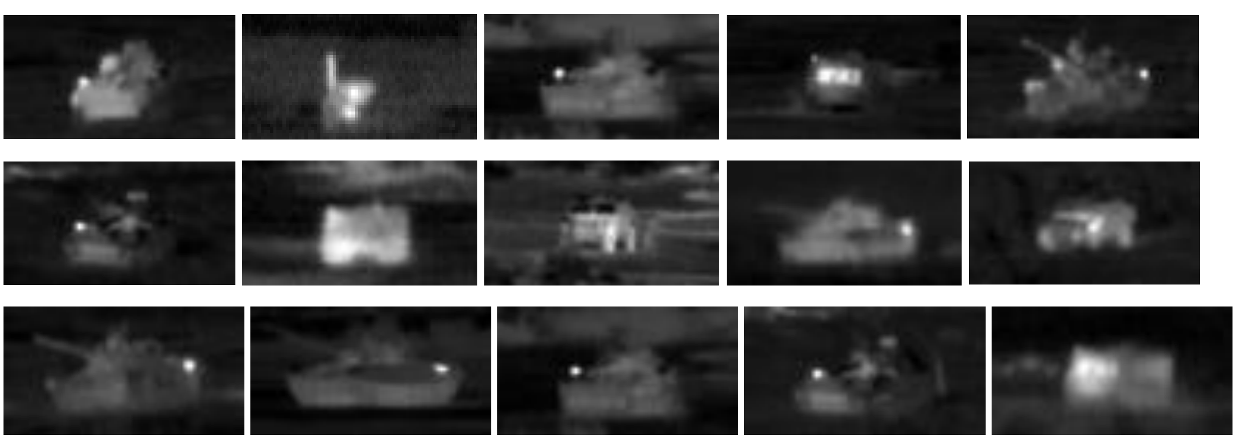


Figure 5.3.2. Middle 25 images of the third dimension of PCA

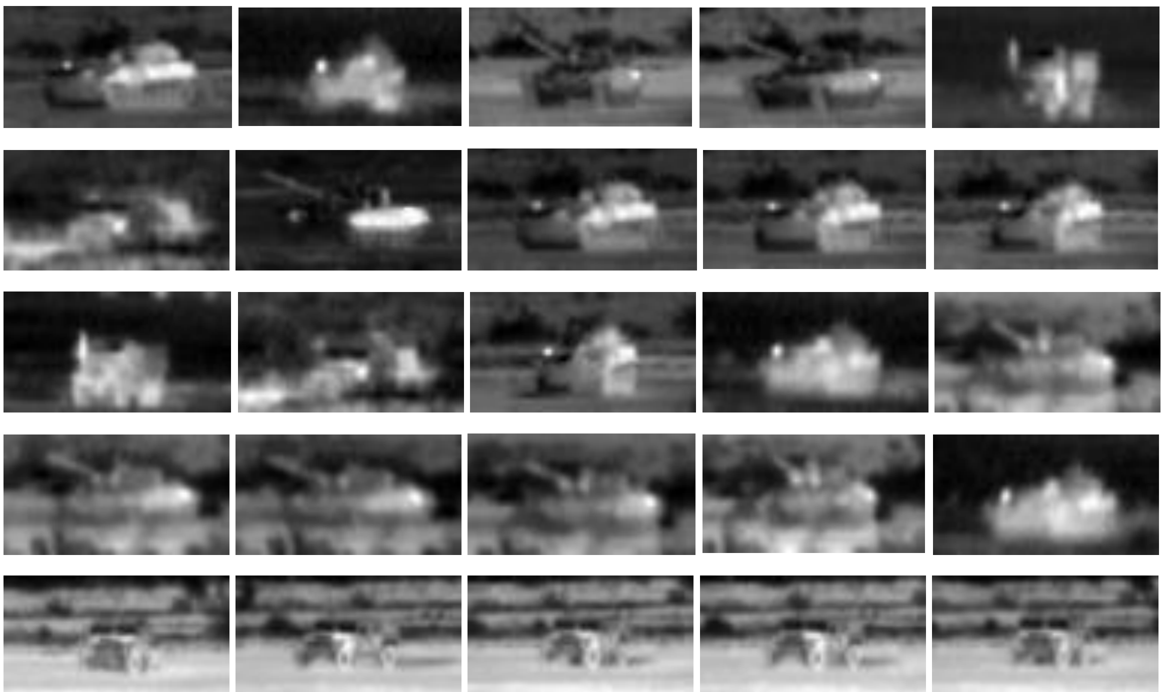
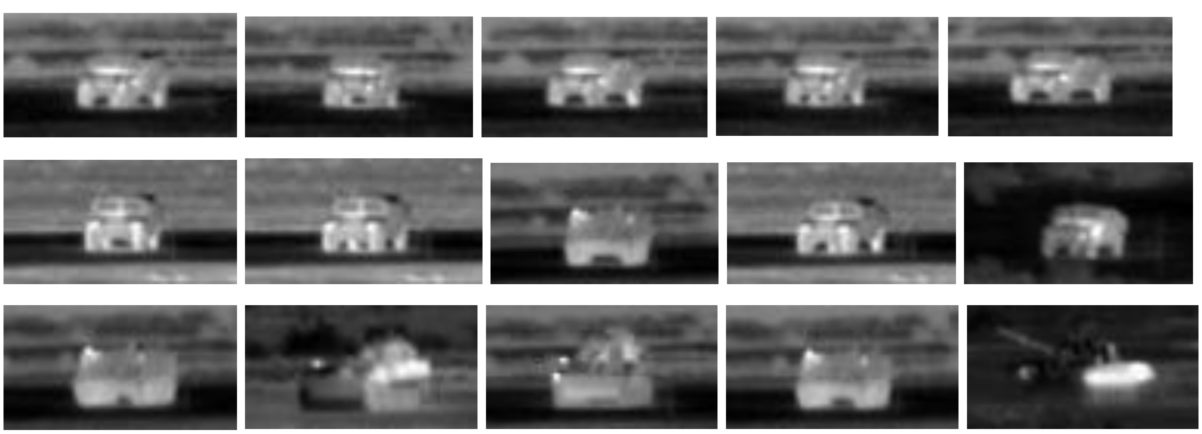


Figure 5.3.3. Last 25 images of the third dimension of PCA



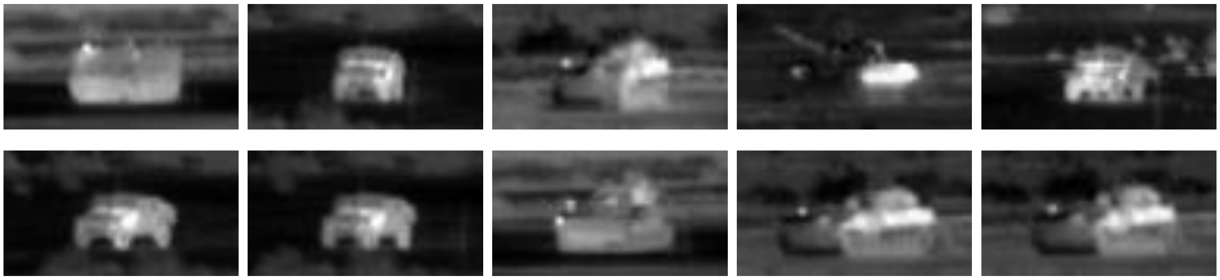


Figure 5.4.1. First 25 images of the fourth dimension of PCA

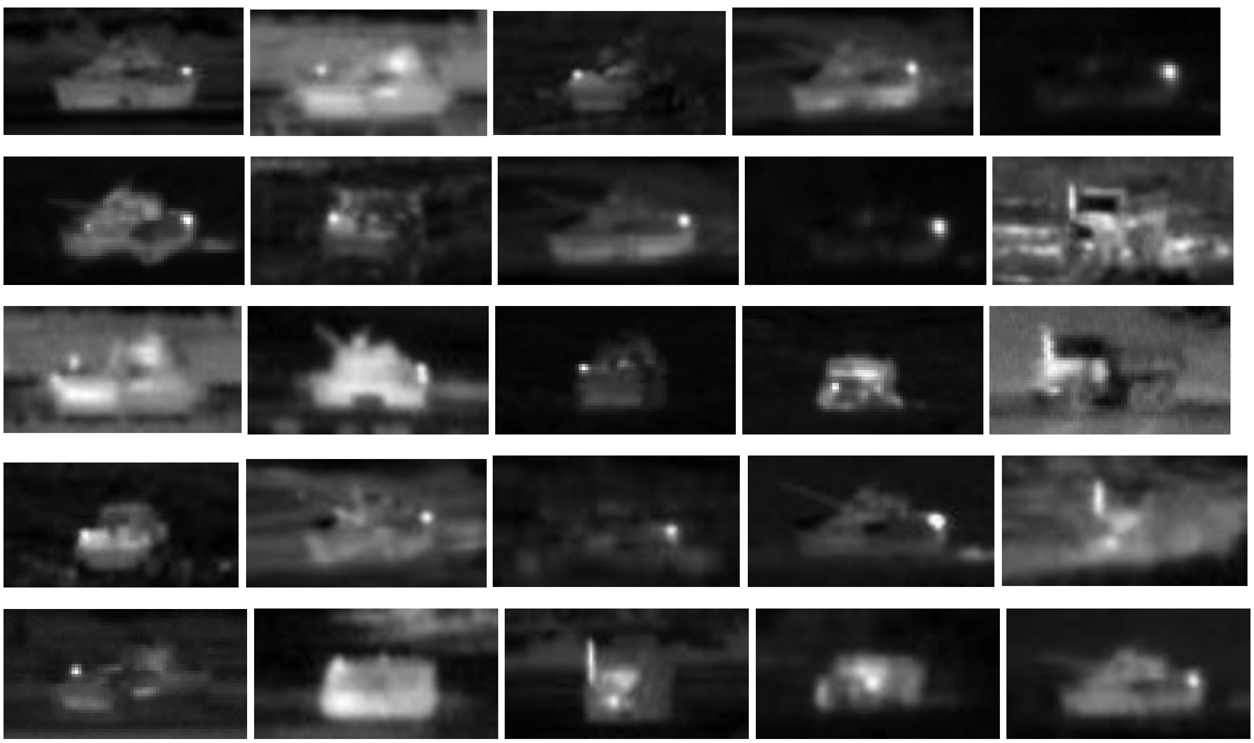
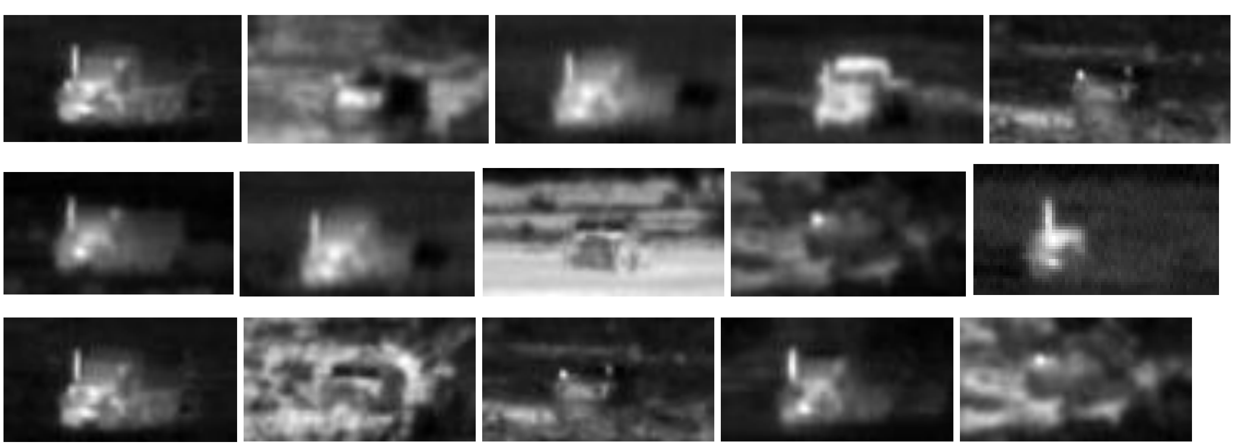


Figure 5.4.2. Middle 25 images of the fourth dimension of PCA



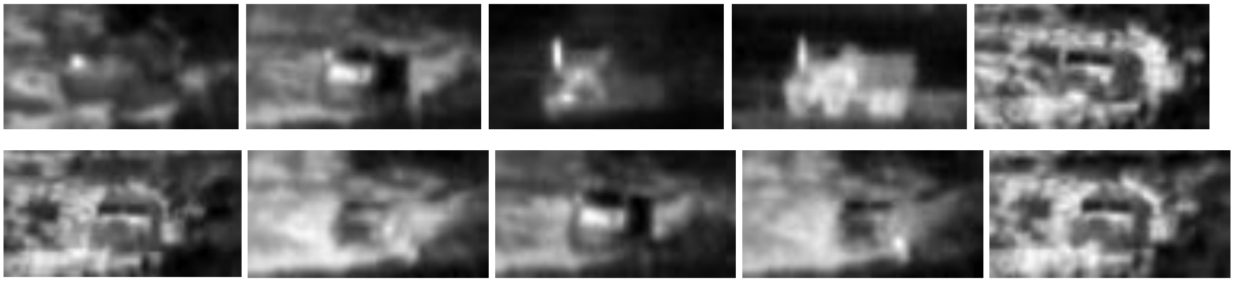


Figure 5.4.3. Last 25 images of the fourth dimension of PCA

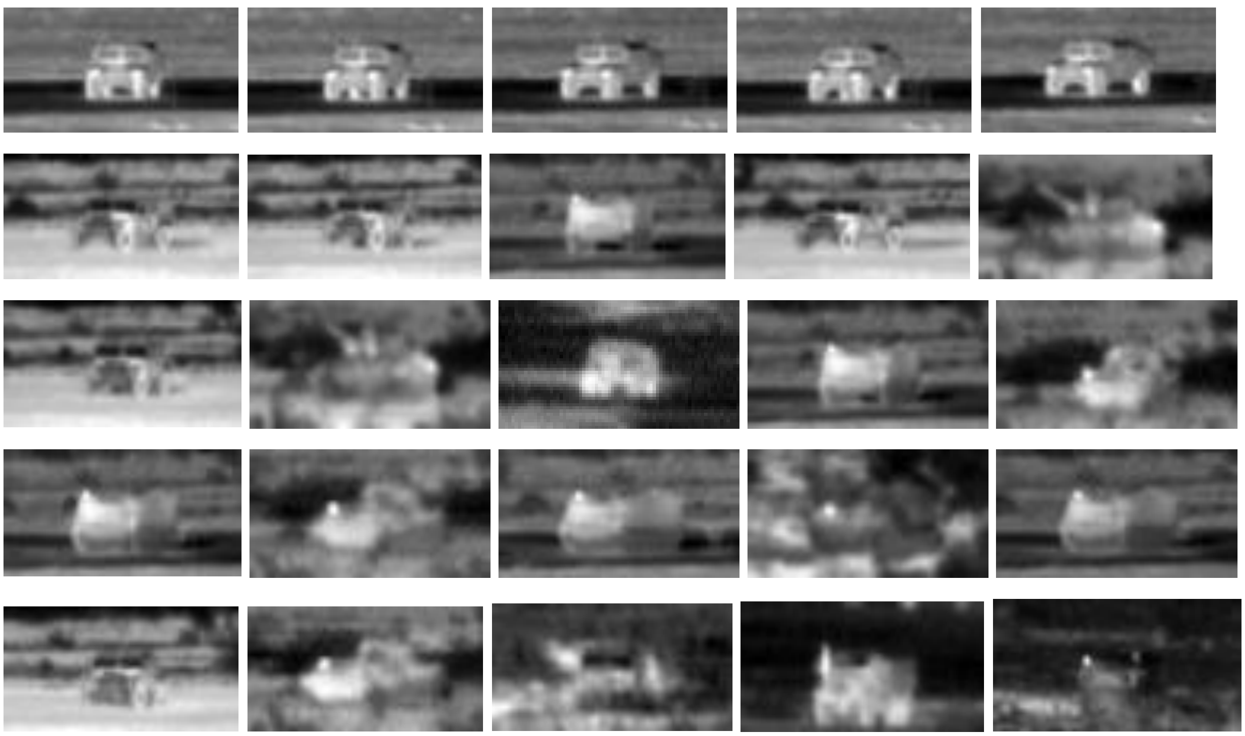
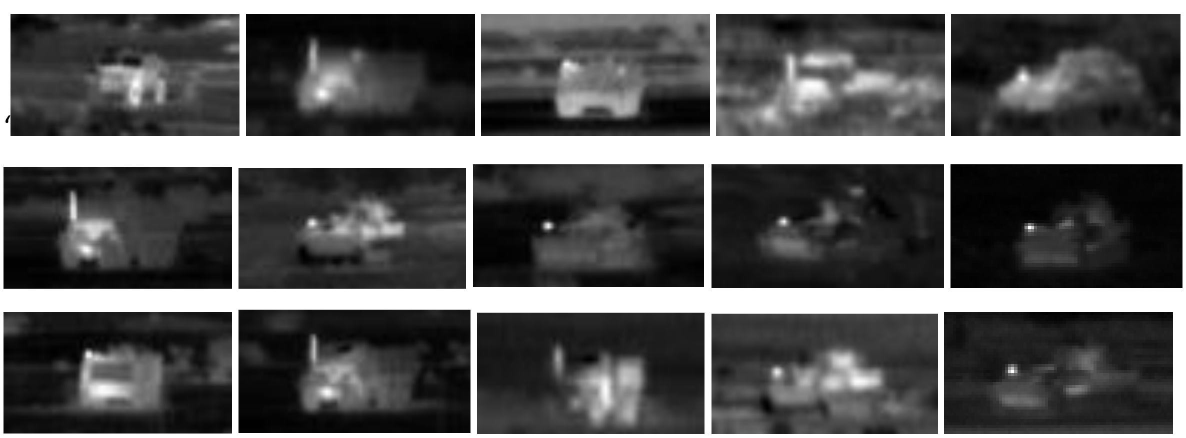


Figure 5.5.1. First 25 images of the fifth dimension of PCA



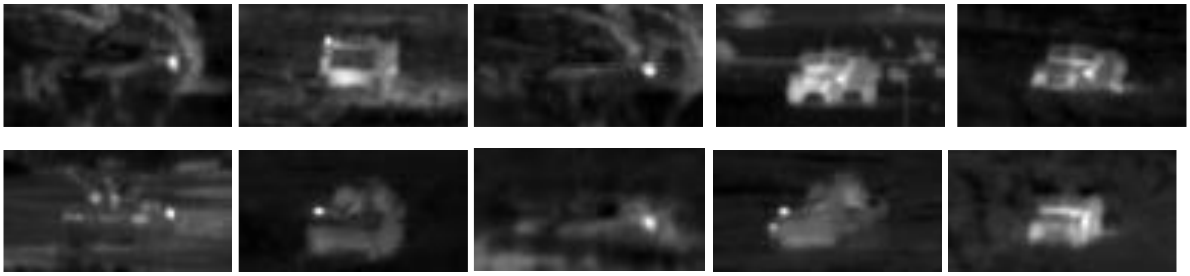


Figure 5.5.2. Middle 25 images of the fifth dimension of PCA

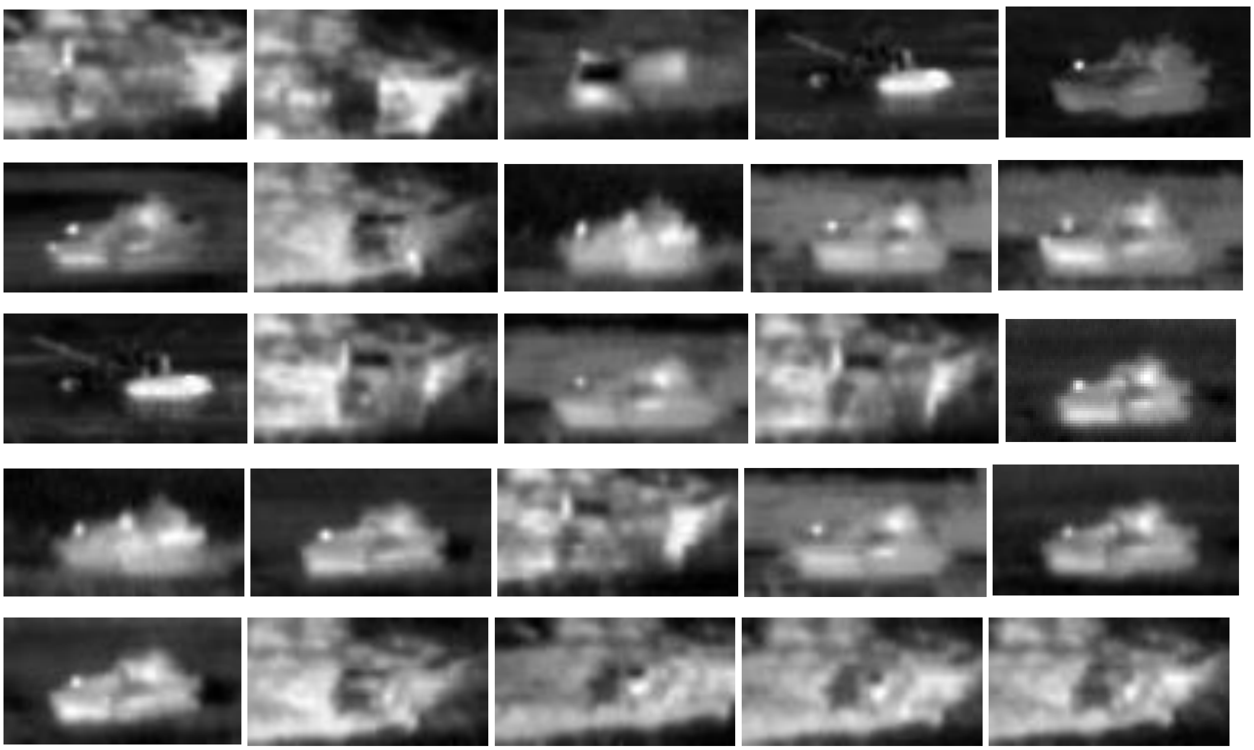
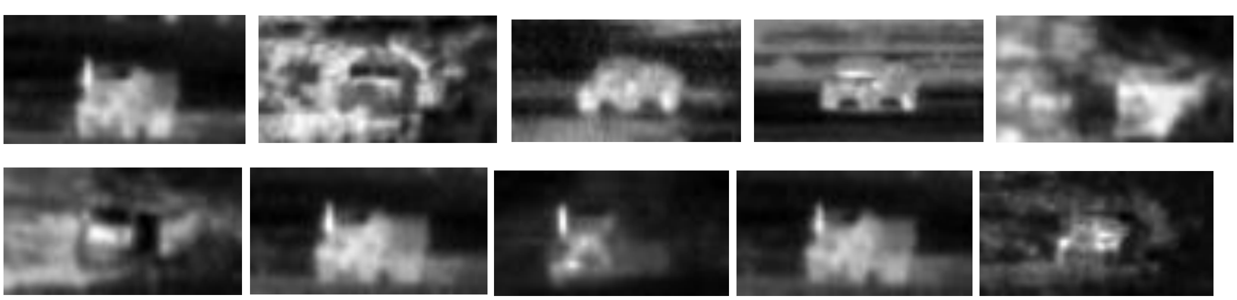


Figure 5.5.3. Last 25 images of the fifth dimension of PCA

MDS



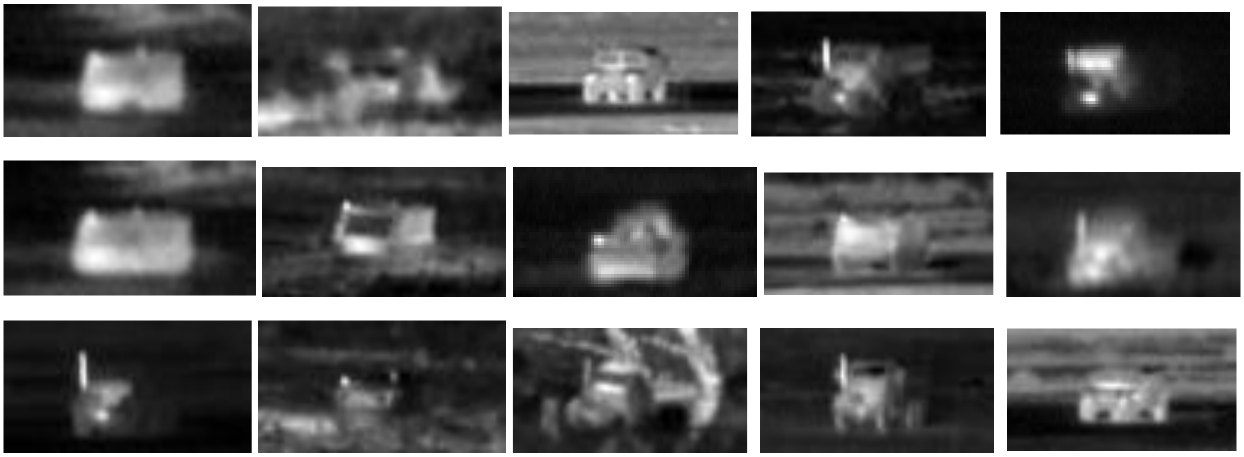


Figure 6.1.1. First 25 images of the first dimension of MDS

Figure 6.1.1. Middle 25 images of the first dimension of MDS

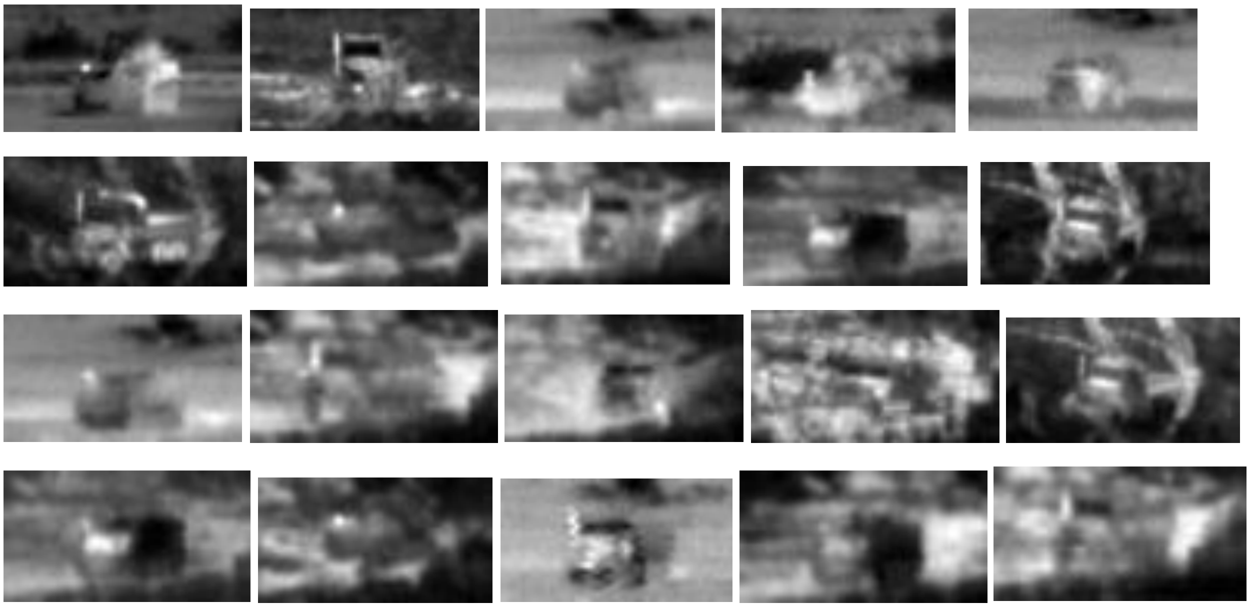


Figure 6.1.1. Last 25 images of the first dimension of MDS

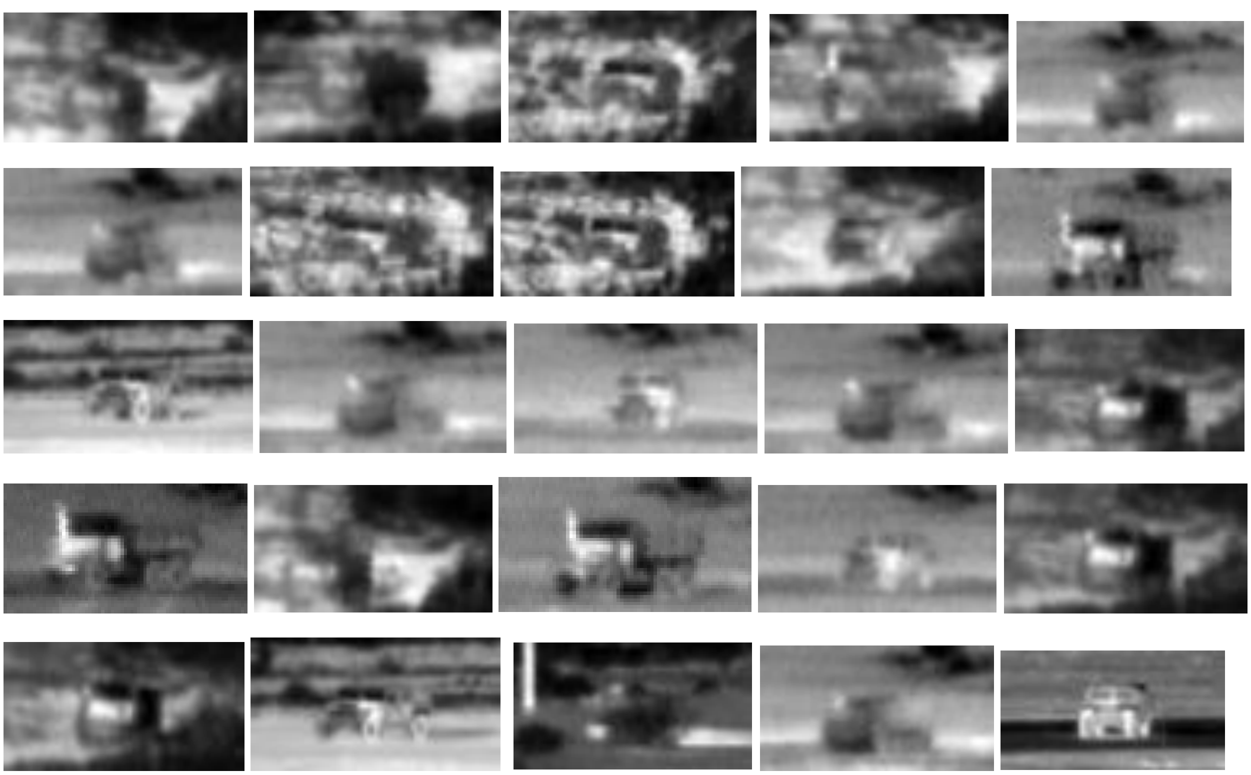


Figure 6.2.1. First 25 images of the second dimension of MDS

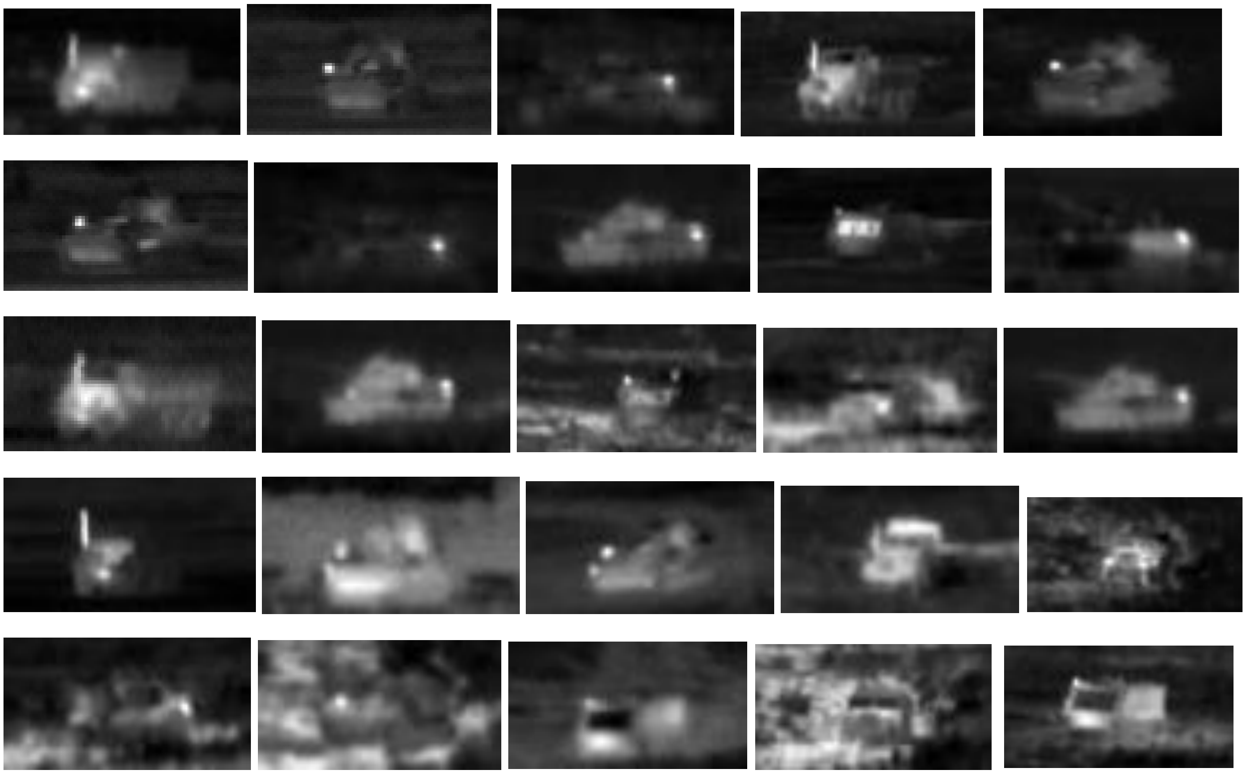


Figure 6.2.2. Middle 25 images of the second dimension of MDS

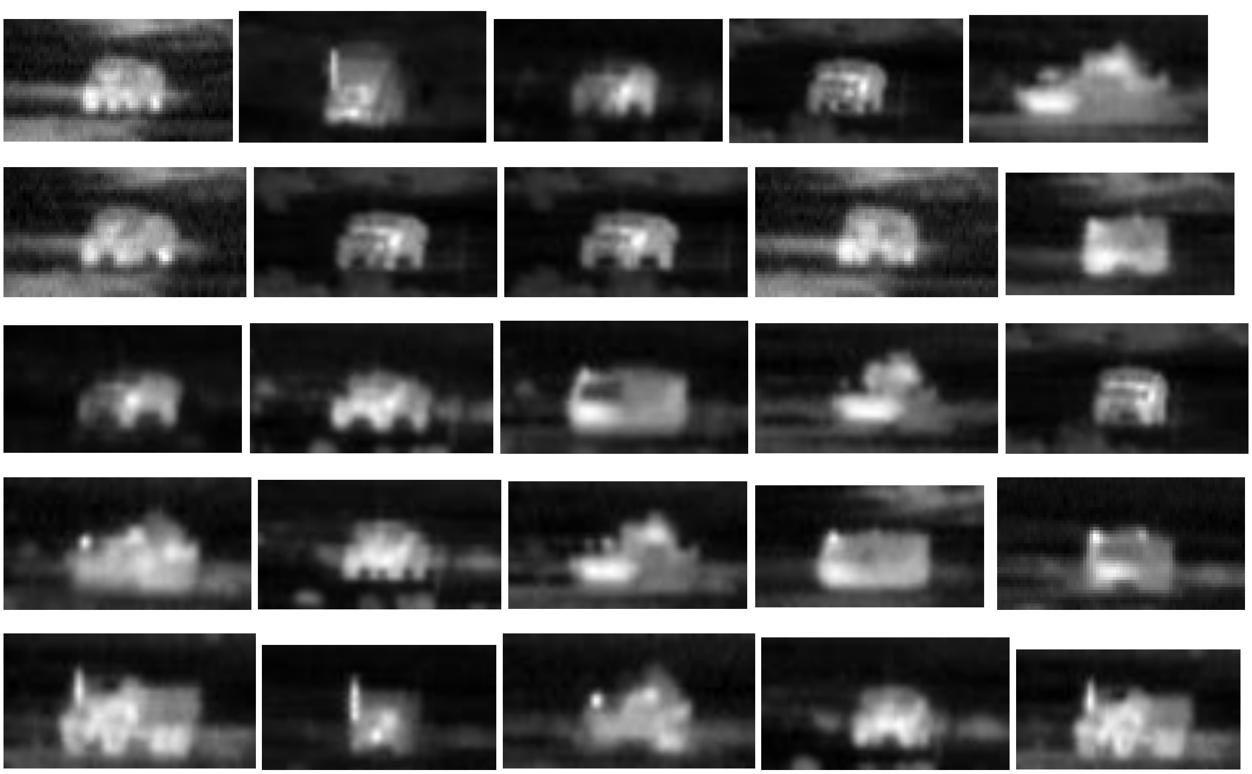


Figure 6.2.3. Last 25 images of the second dimension of MDS

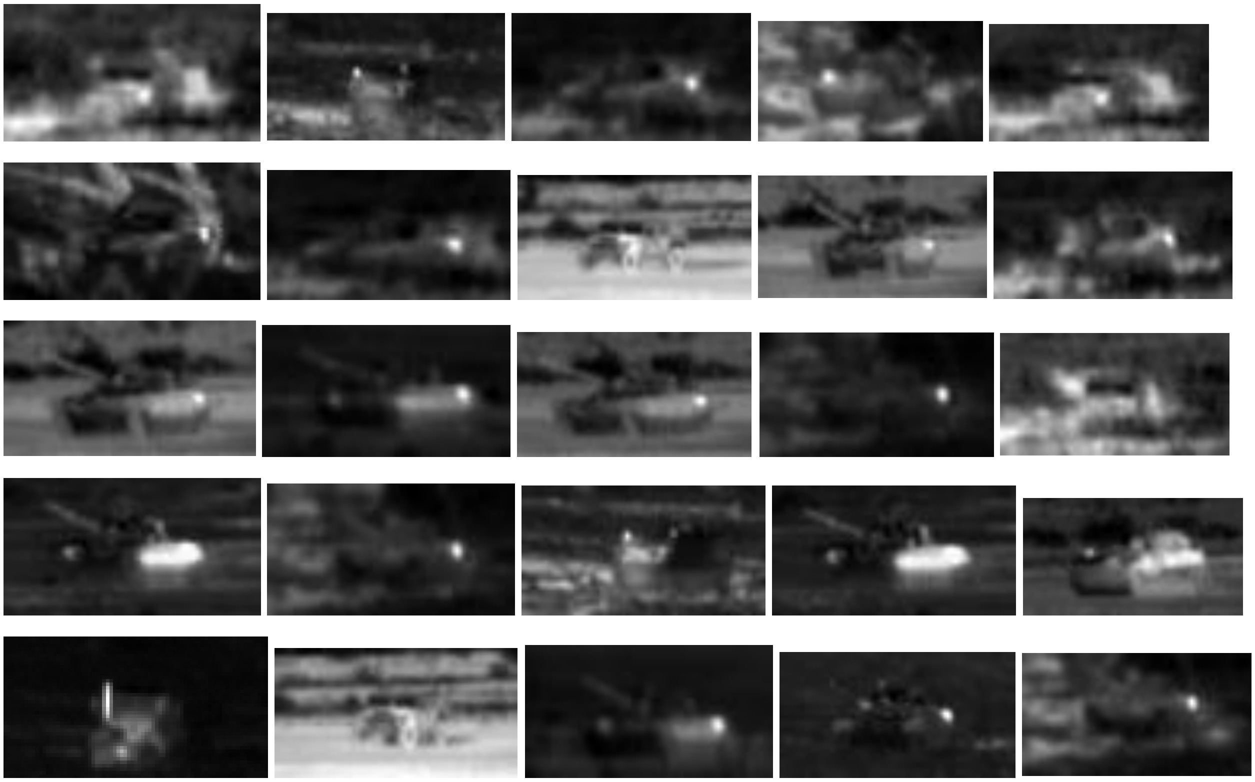


Figure 6.3.1. First 25 images of the third dimension of MDS

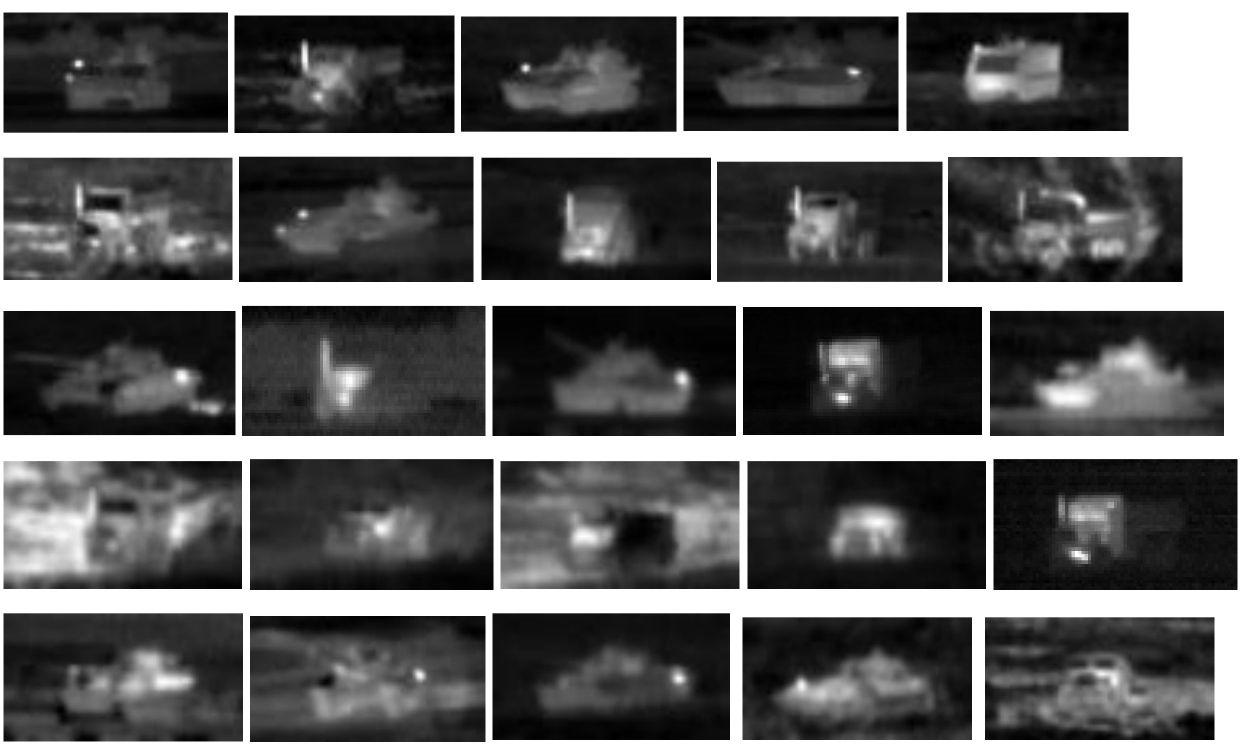


Figure 6.3.2. Middle 25 images of the third dimension of MDS

Figure 6.3.3. Last 25 images of the third dimension of MDS

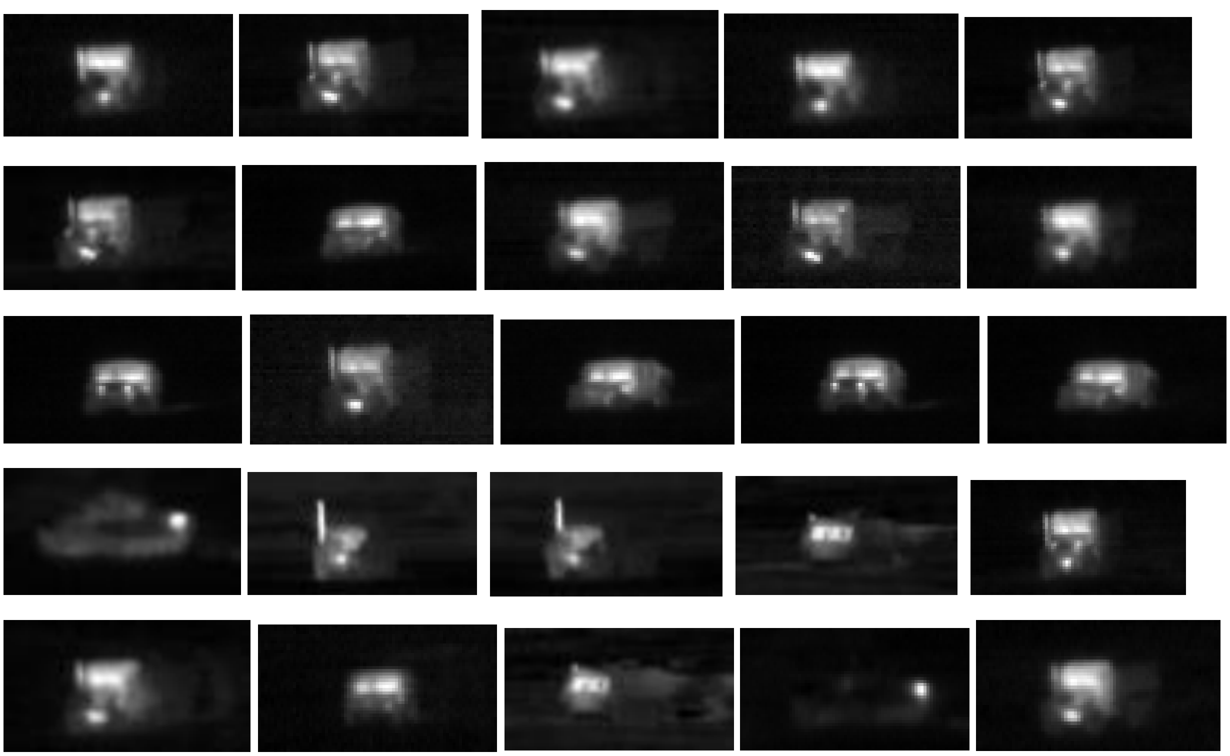


Figure 6.4.1. Last 25 images of the fourth dimension of MDS

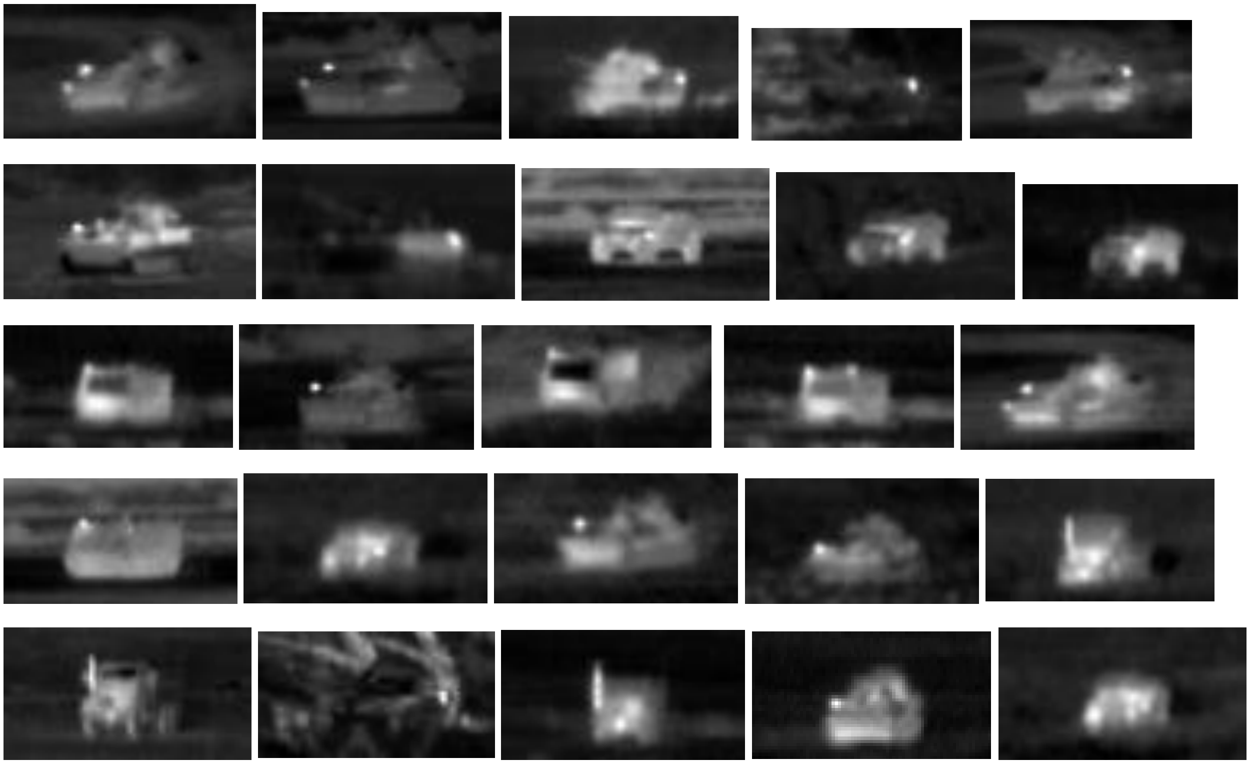


Figure 6.4.2. Middle 25 images of the fourth dimension of MDS

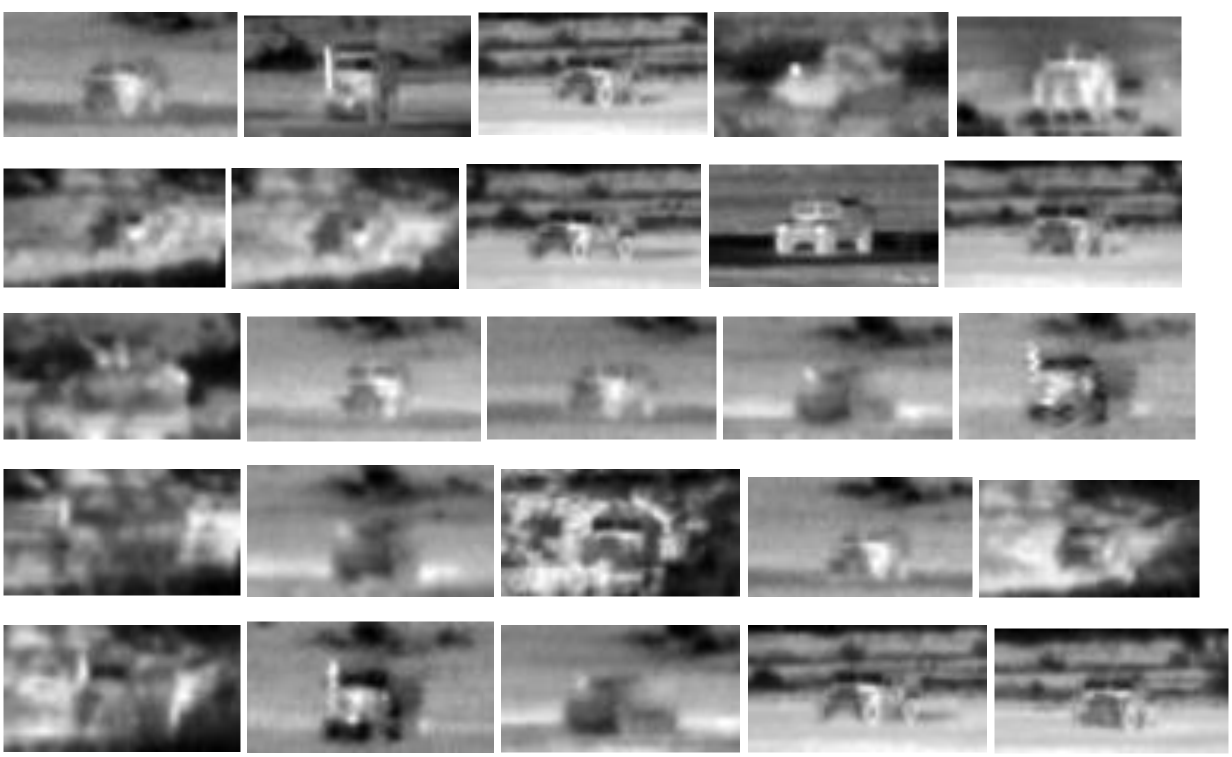


Figure 6.4.3. Last 25 images of the fourth dimension of MDS

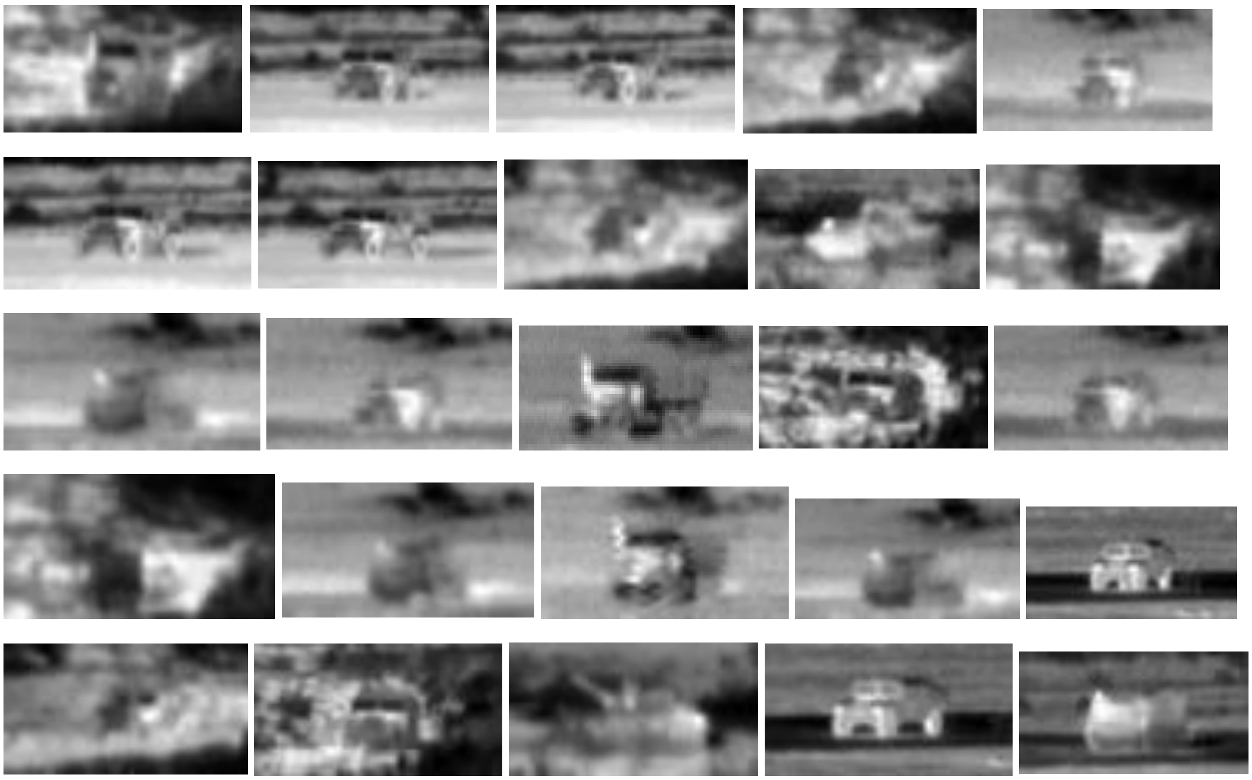
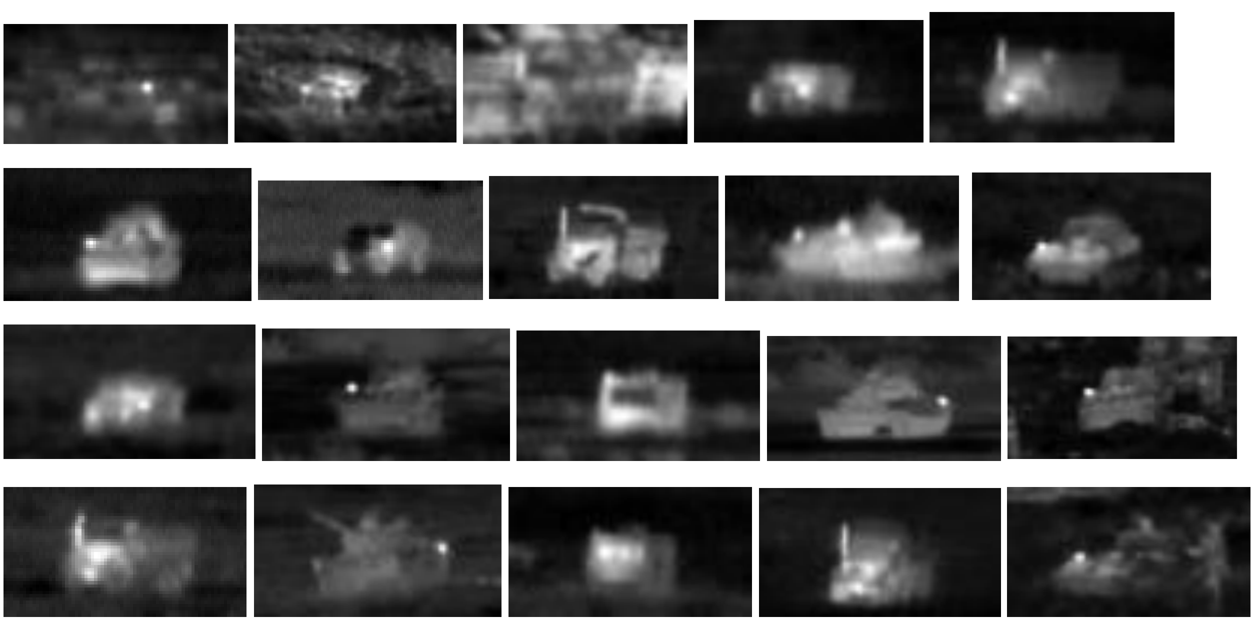


Figure 6.5.1. First 25 images of the fifth dimension of MDS



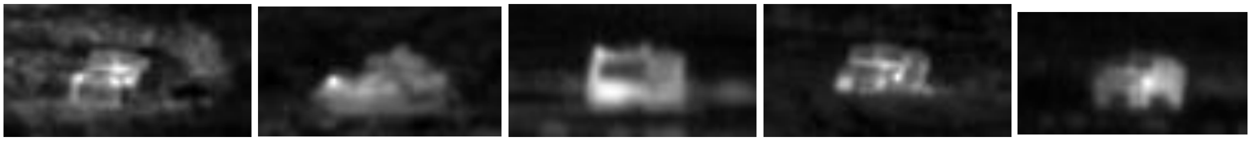


Figure 6.5.2. Middle 25 images of the fifth dimension of MDS

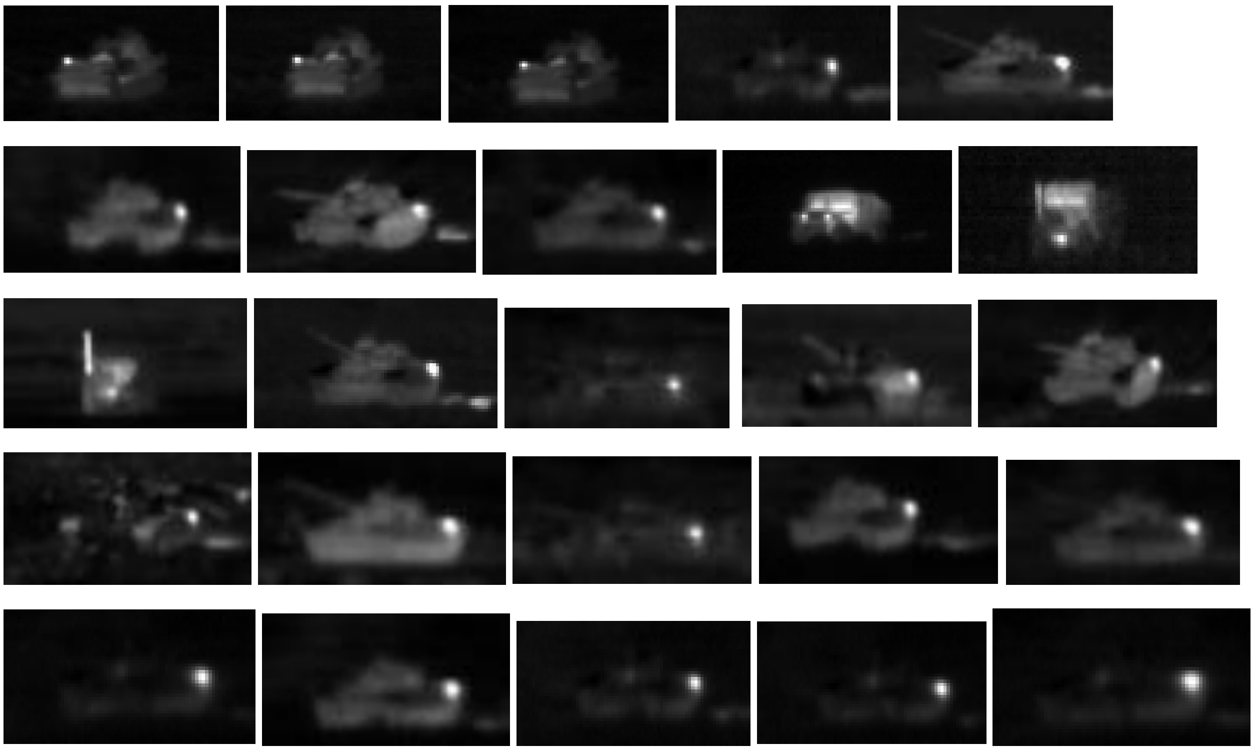
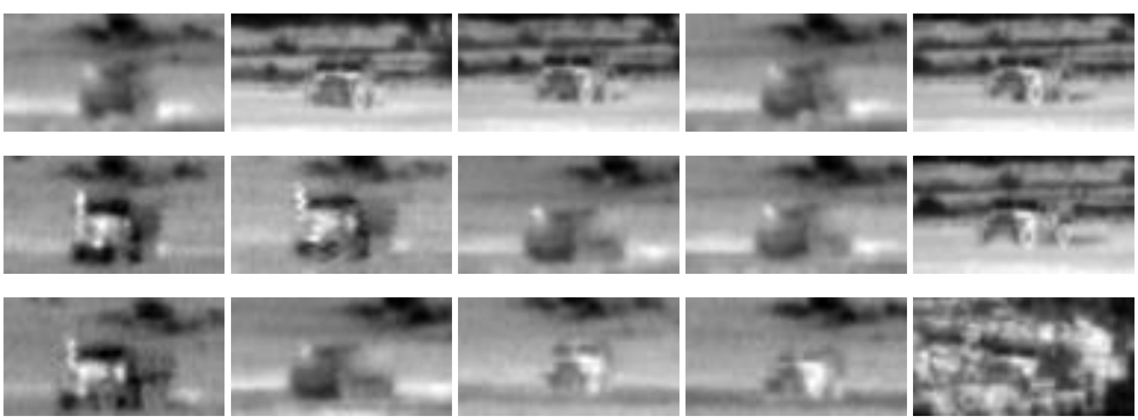


Figure 6.5.3. Last 25 images of the fifth dimension of MDS

Isomap



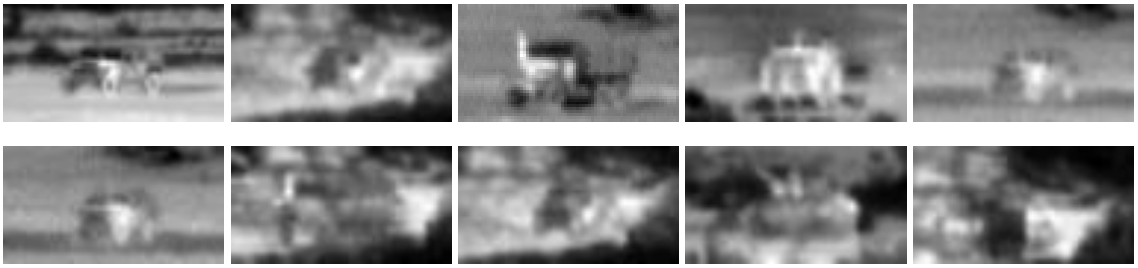


Figure 7.1.1. First 25 images of the first dimension of Isomap

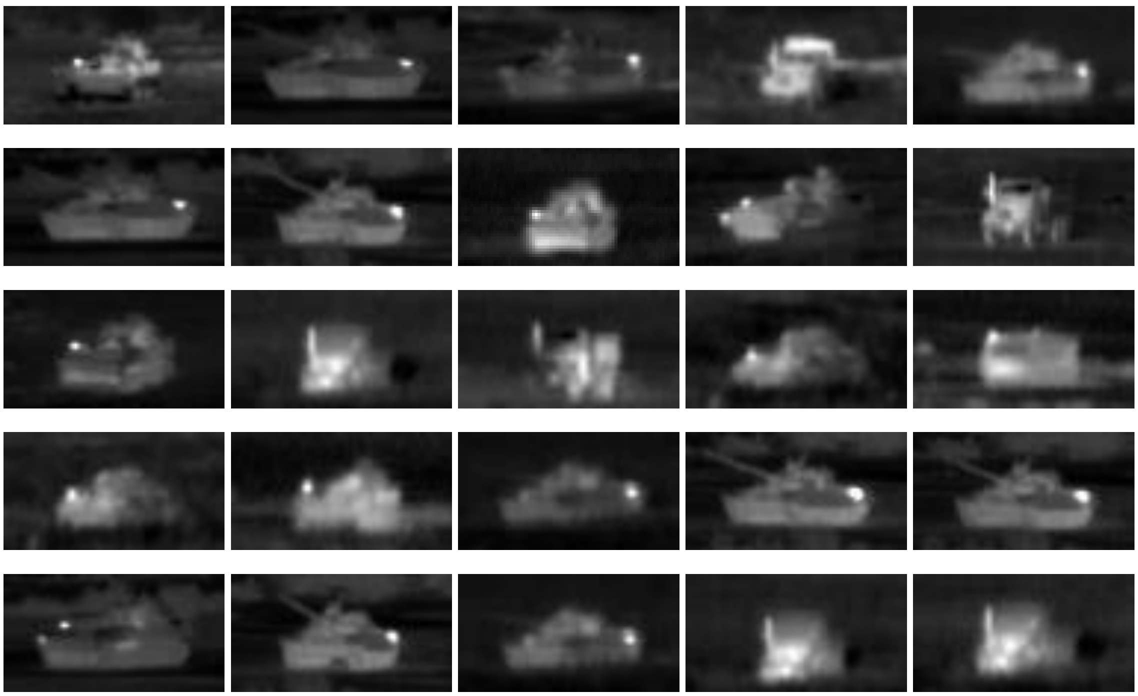
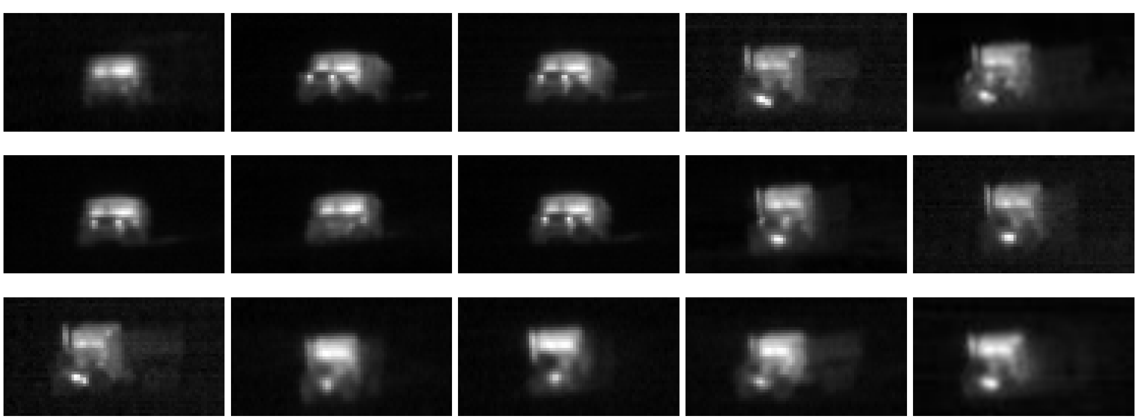


Figure 7.1.2. Middle 25 images of the first dimension of Isomap



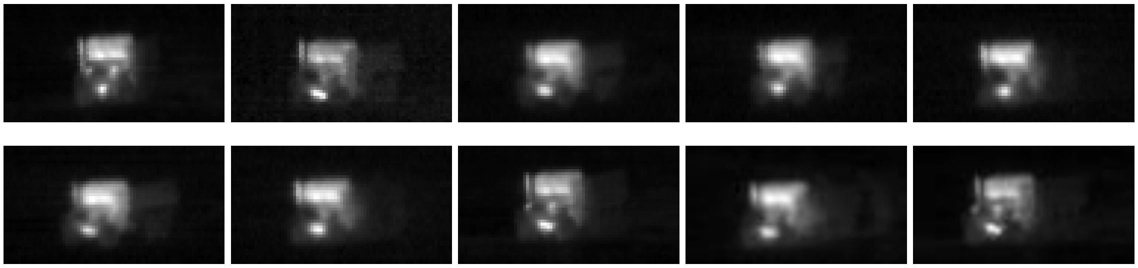


Figure 7.1.3. Last 25 images of the first dimension of Isomap

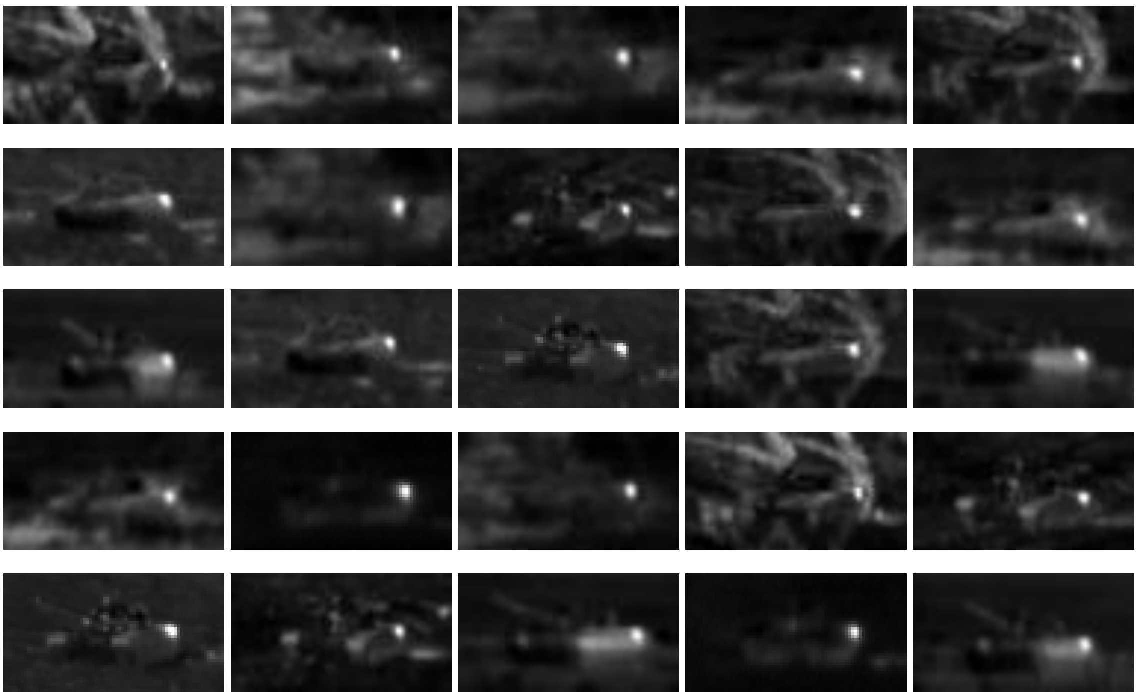
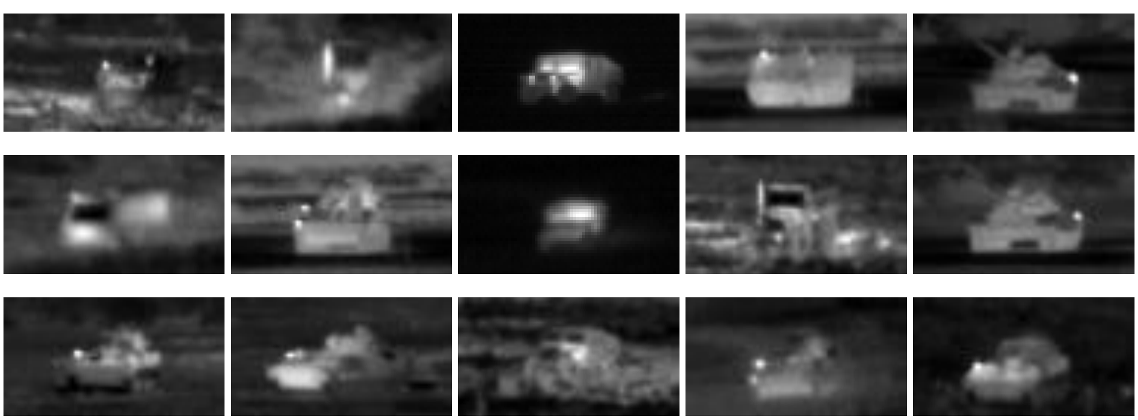


Figure 7.2.1. First 25 images of the second dimension of Isomap



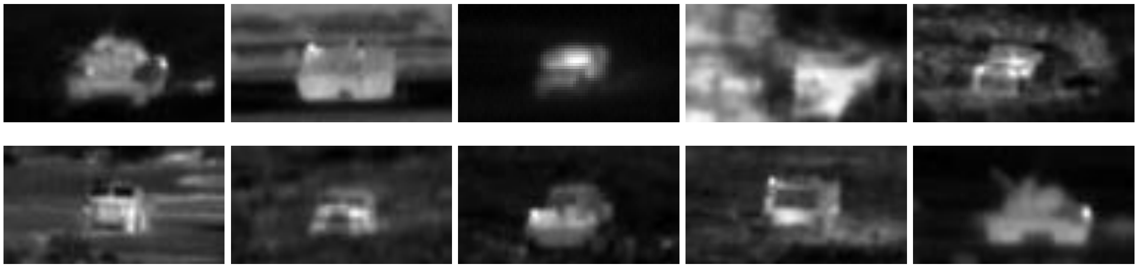


Figure 7.2.2. Middle 25 images of the second dimension of Isomap

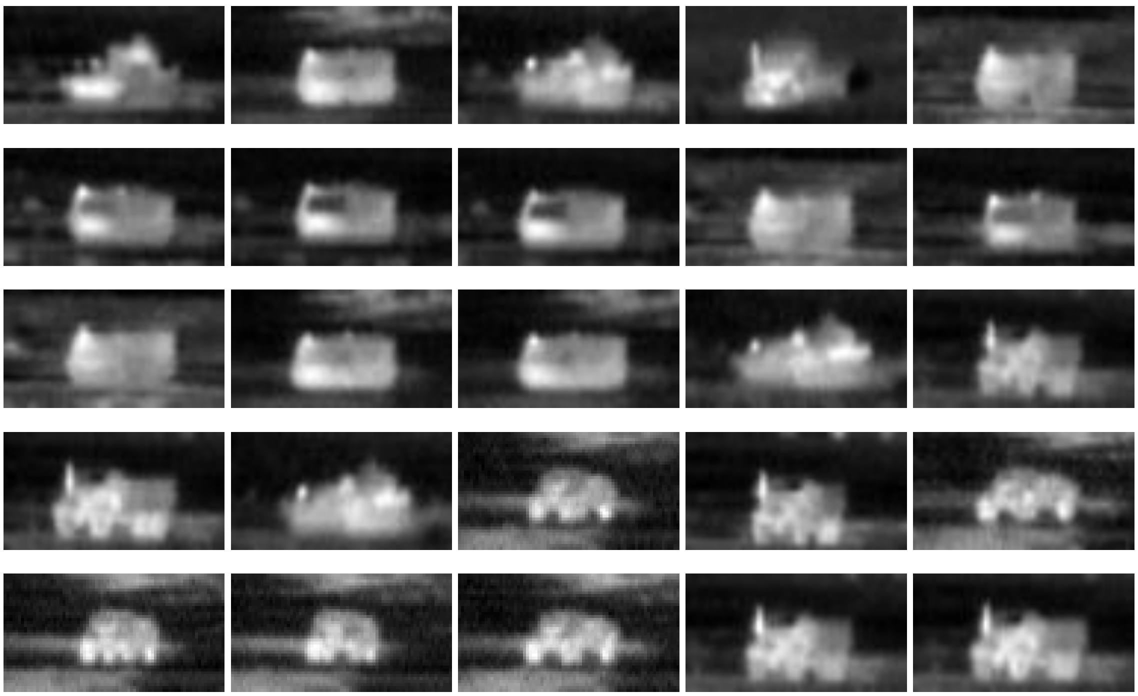


Figure 7.2.3. Last 25 images of the second dimension of Isomap

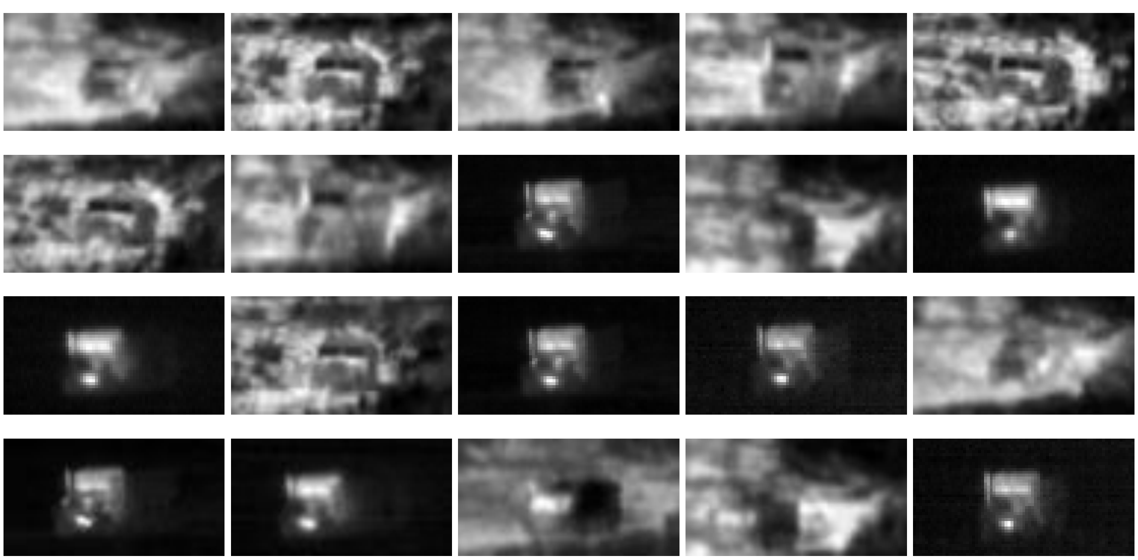


Figure 7.3.1. First 25 images of the third dimension of Isomap

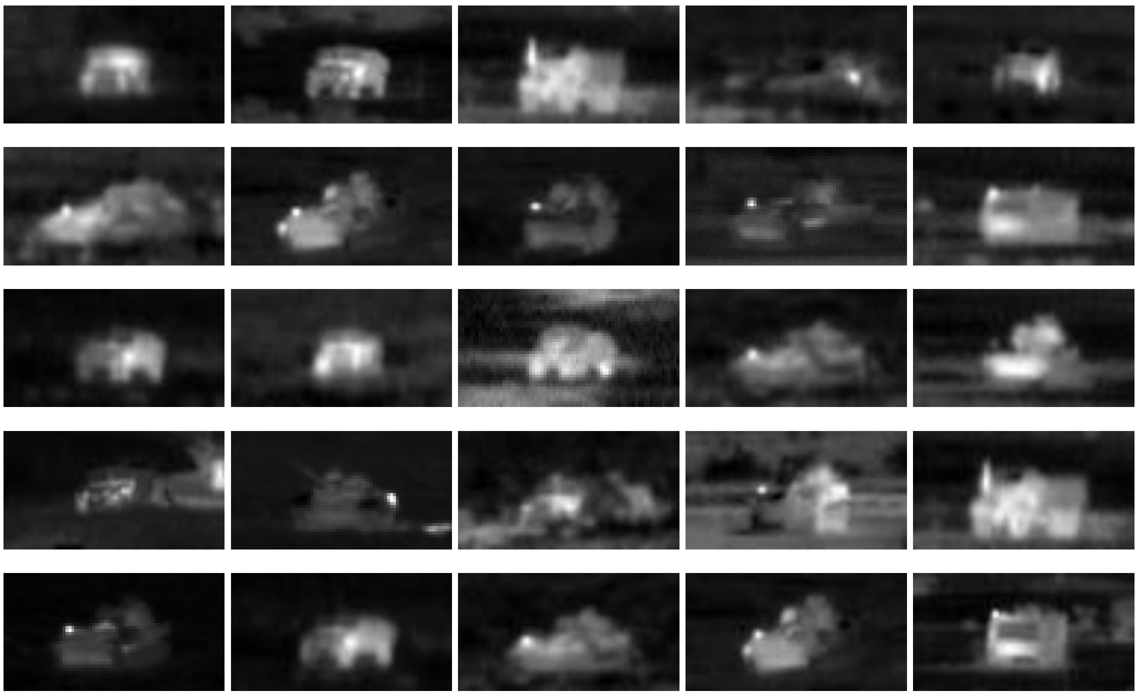


Figure 7.3.2. Middle 25 images of the third dimension of Isomap

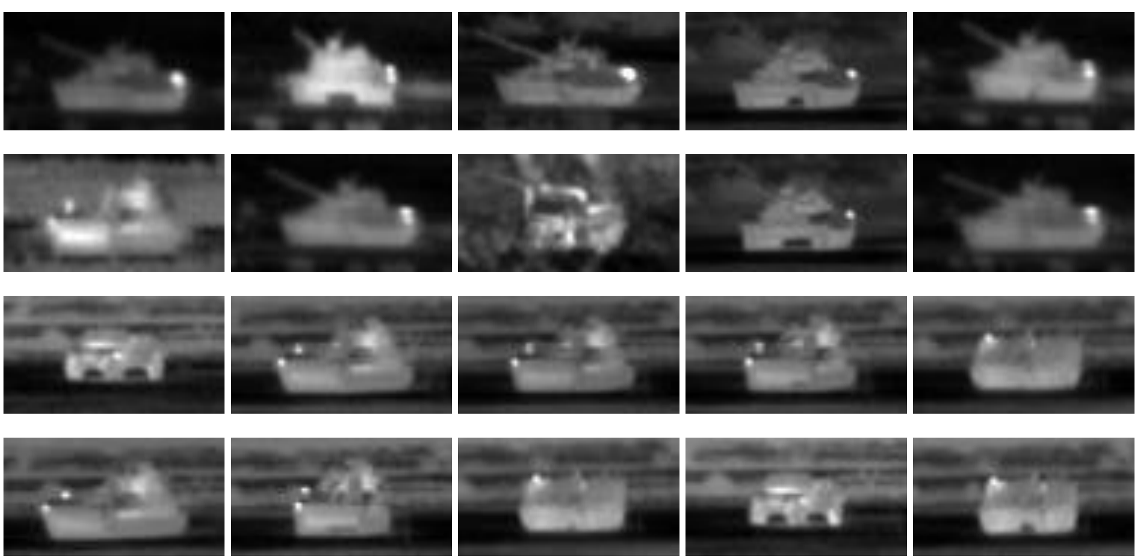


Figure 7.3.3. Last 25 images of the third dimension of Isomap

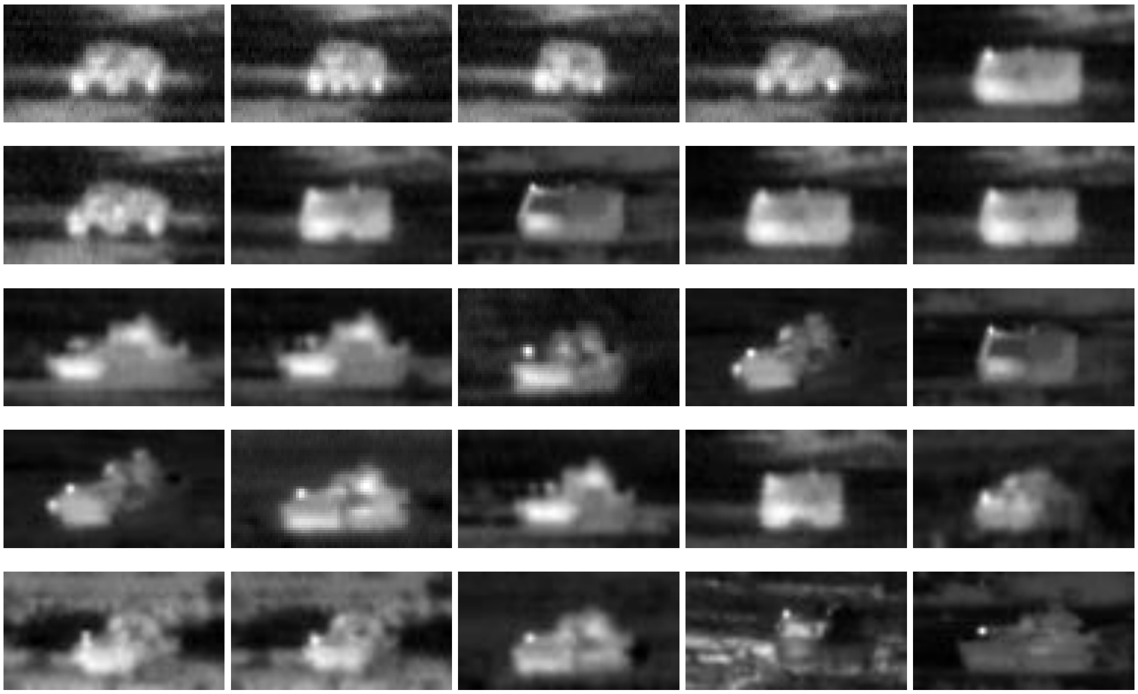


Figure 7.4.1. First 25 images of the fourth dimension of Isomap

Figure 7.4.2. Middle 25 images of the fourth dimension of Isomap

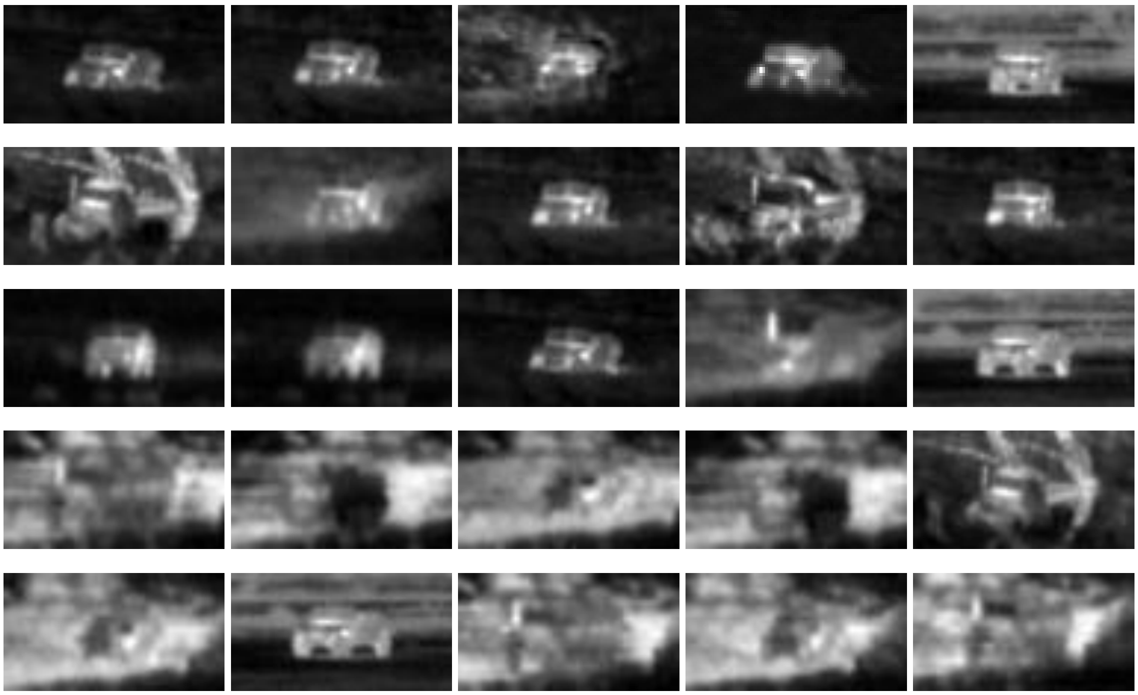
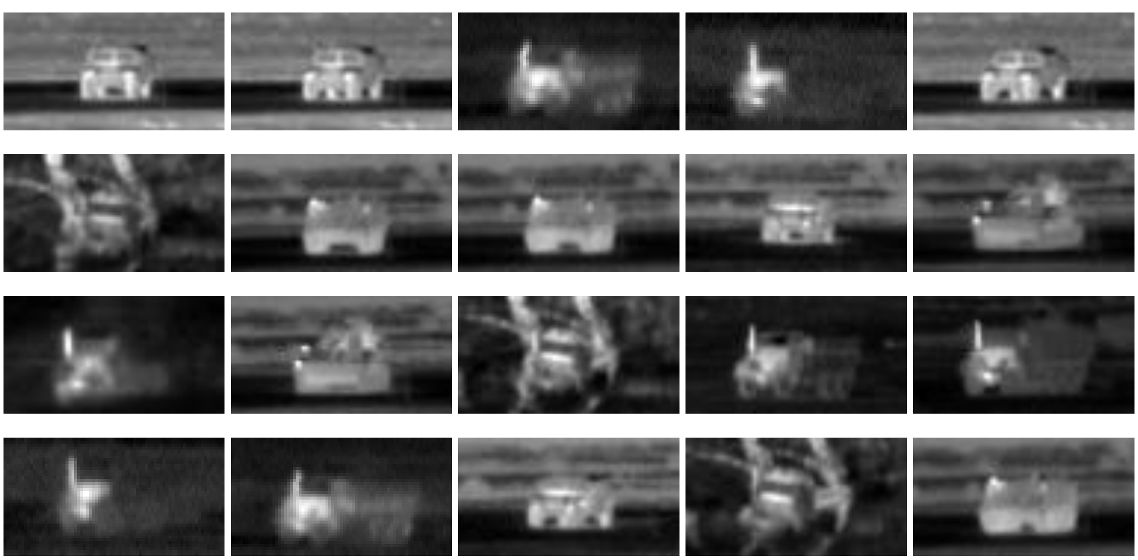


Figure 7.4.3. Last 25 images of the fourth dimension of Isomap



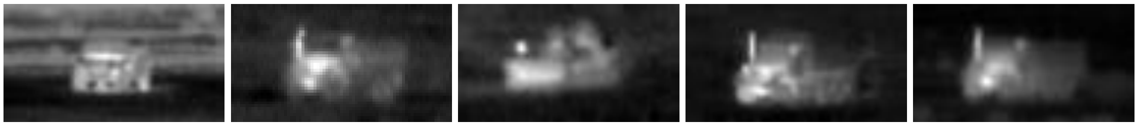


Figure 7.5.1. First 25 images of the fifth dimension of Isomap

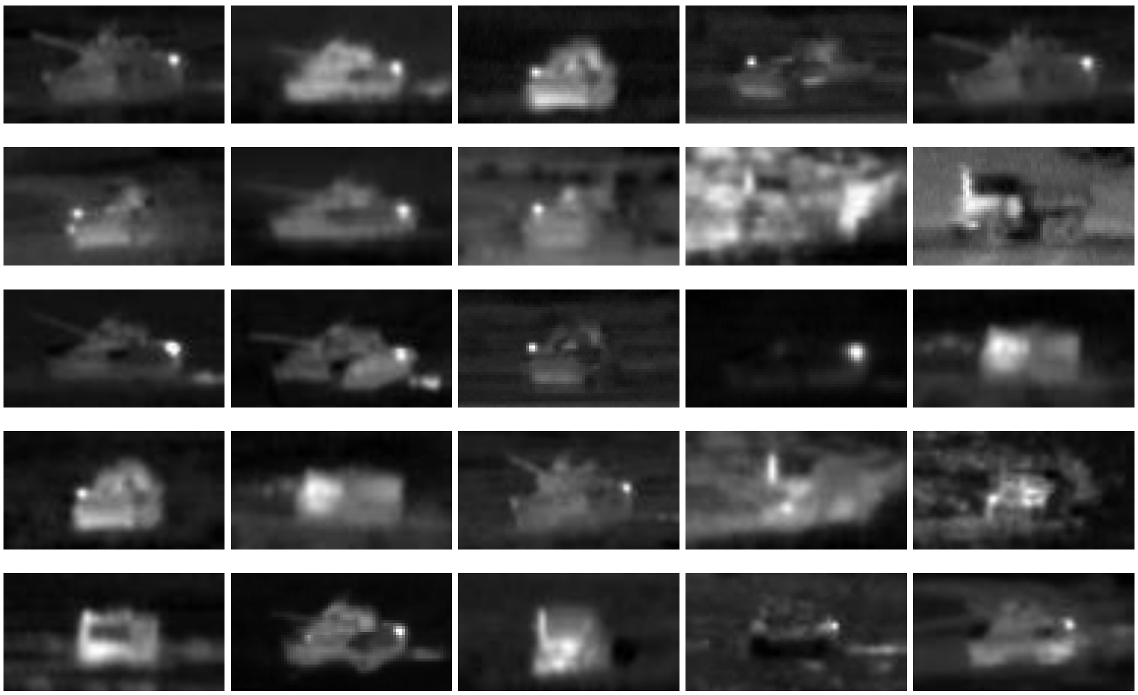


Figure 7.5.2. Middle 25 images of the fifth dimension of Isomap

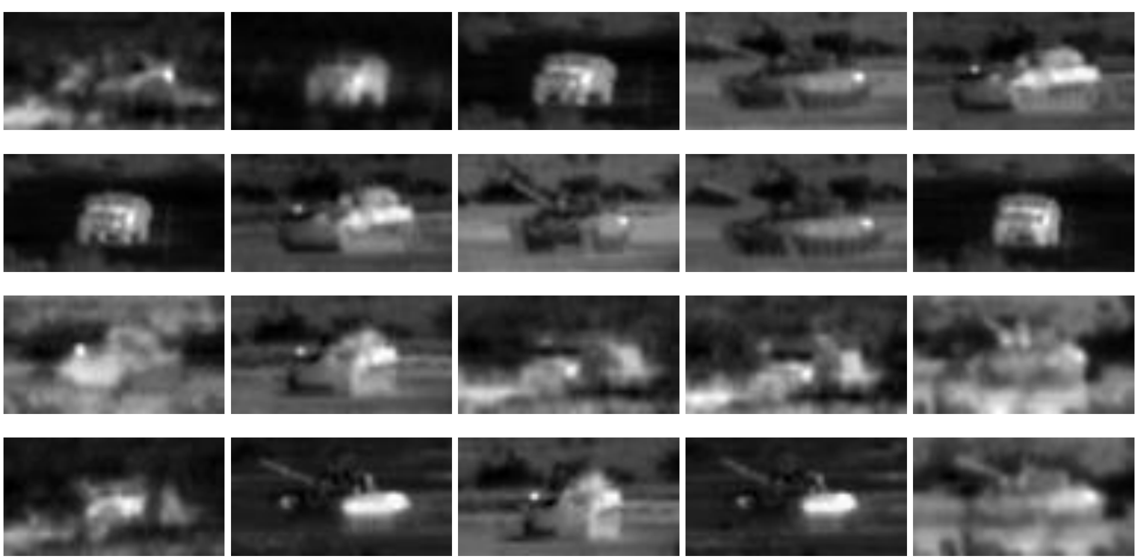


Figure 7.5.3. Last 25 images of the fifth dimension of Isomap

Kernel PCA

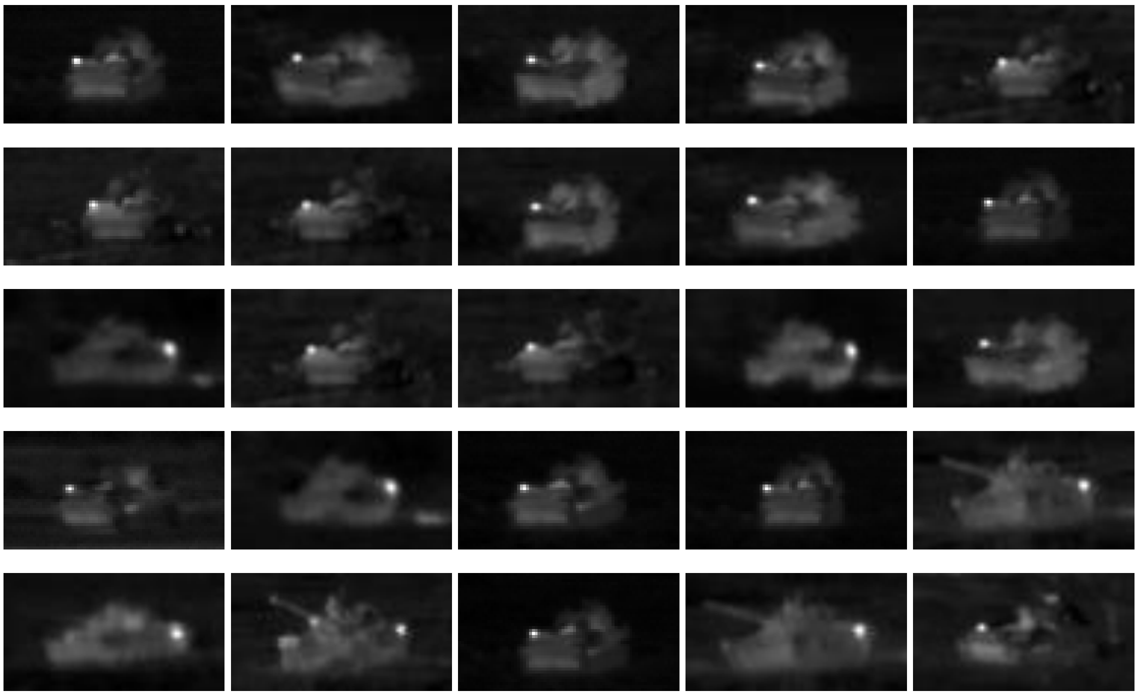


Figure 8.1.1. First 25 images of the first dimension of Kernel PCA

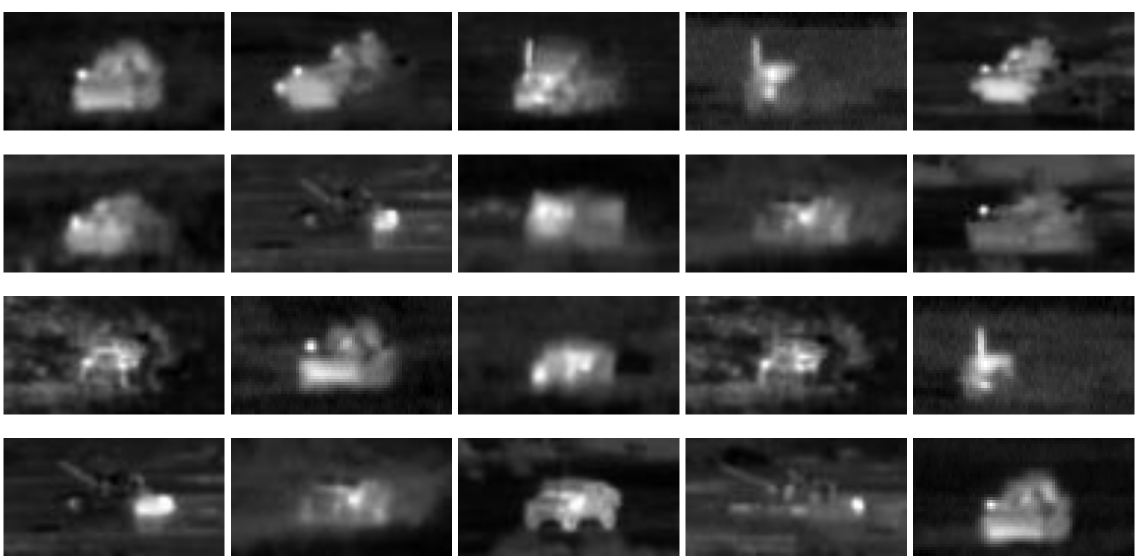


Figure 8.1.2. Middle 25 images of the first dimension of Kernel PCA

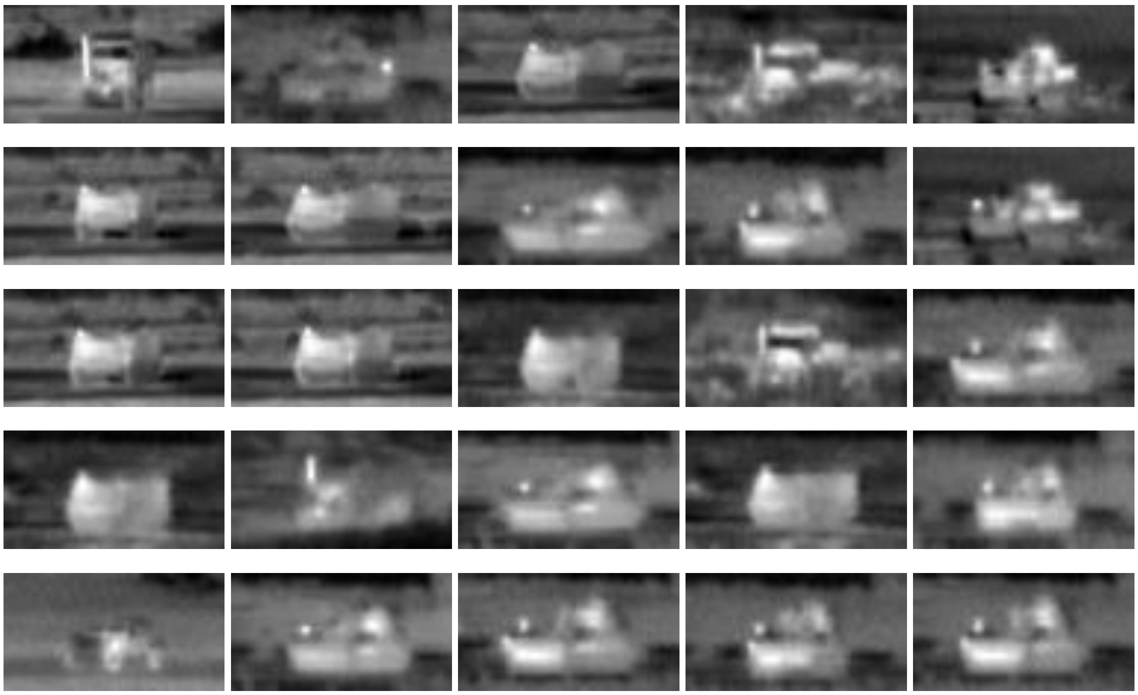


Figure 8.1.3. Last 25 images of the first dimension of Kernel PCA

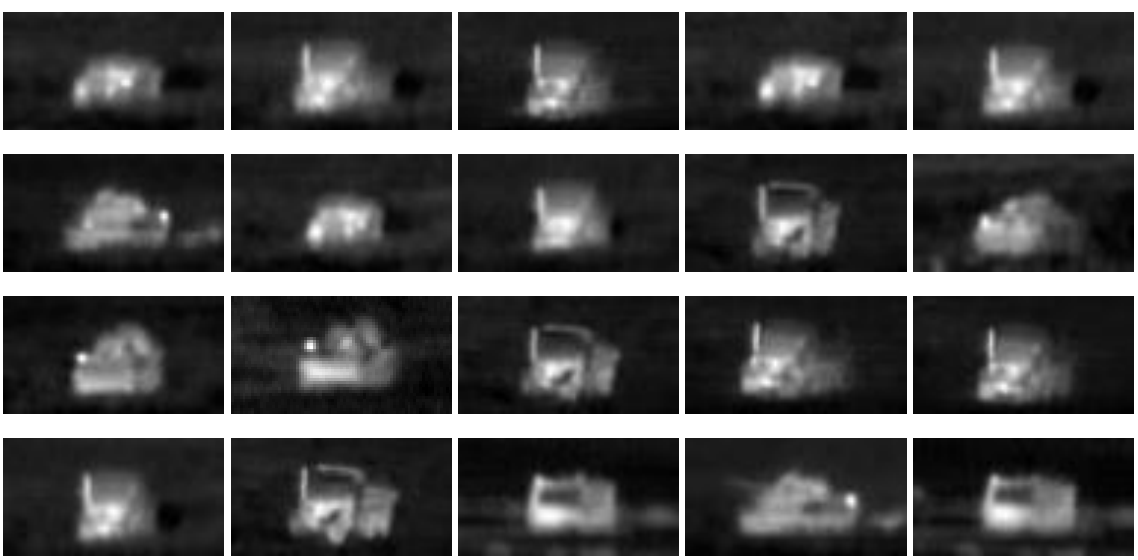


Figure 8.2.1. First 25 images of the second dimension of Kernel PCA

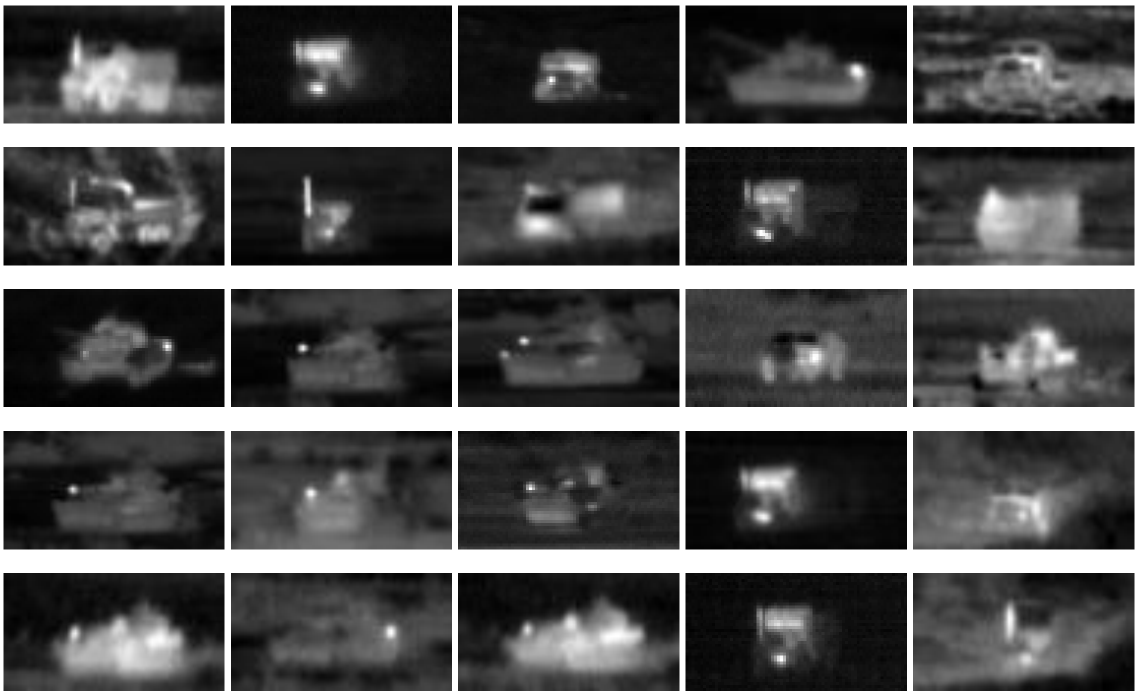
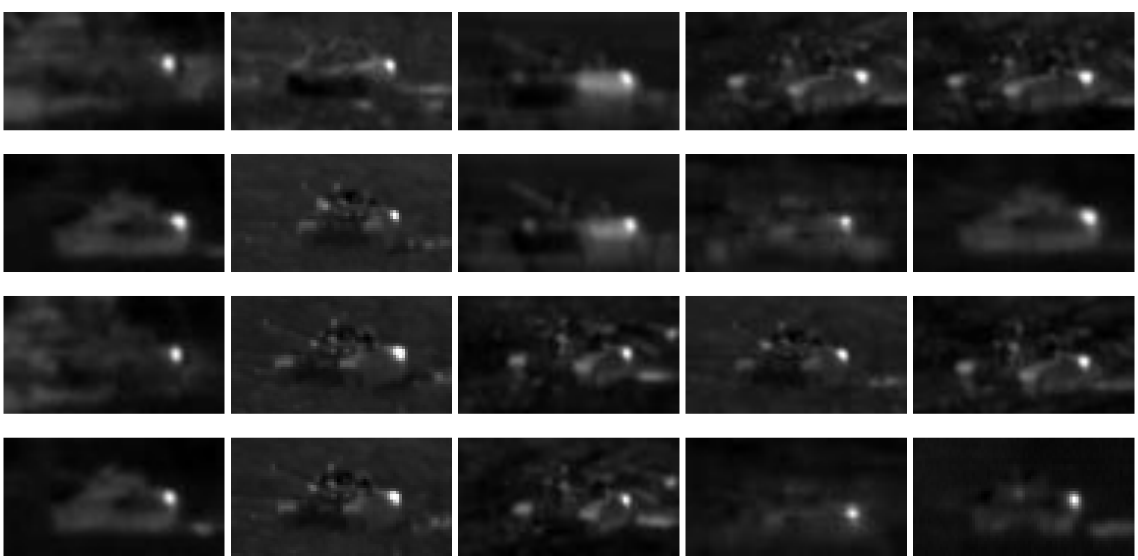


Figure 8.2.2. Middle 25 images of the second dimension of Kernel PCA



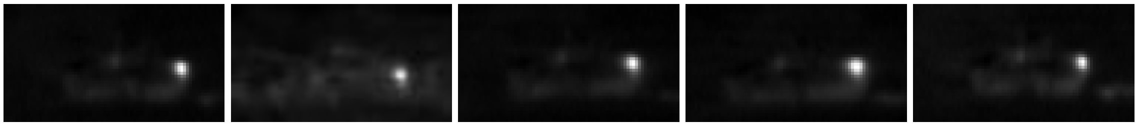


Figure 8.2.3. Last 25 images of the second dimension of Kernel PCA

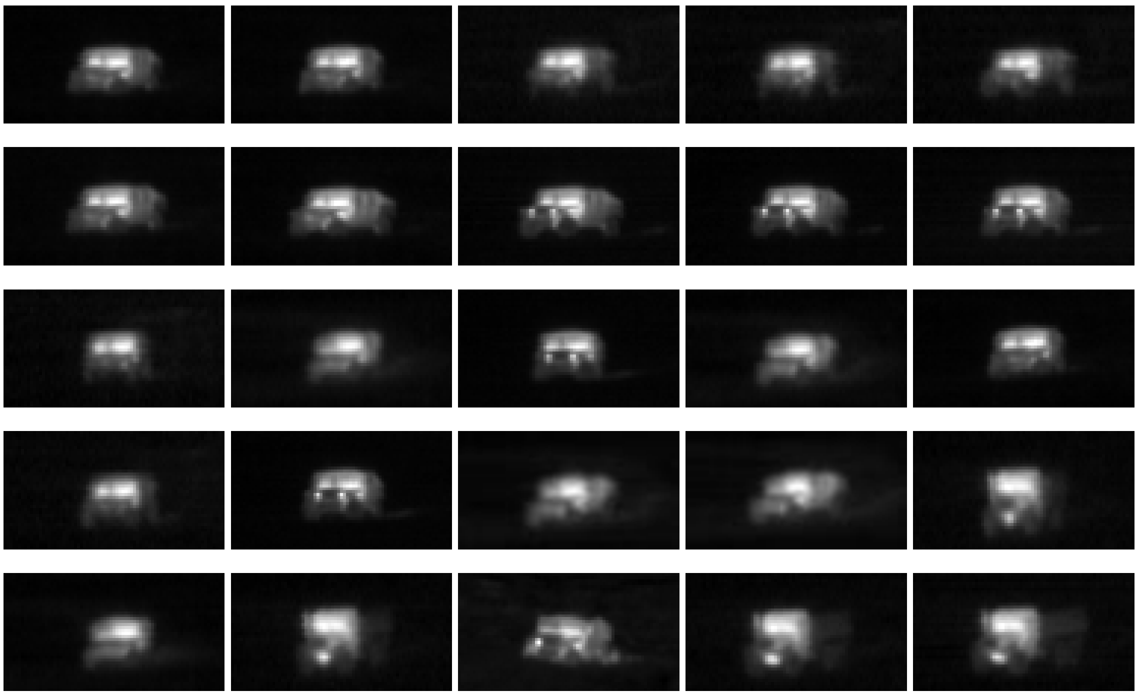


Figure 8.3.1. First 25 images of the third dimension of Kernel PCA

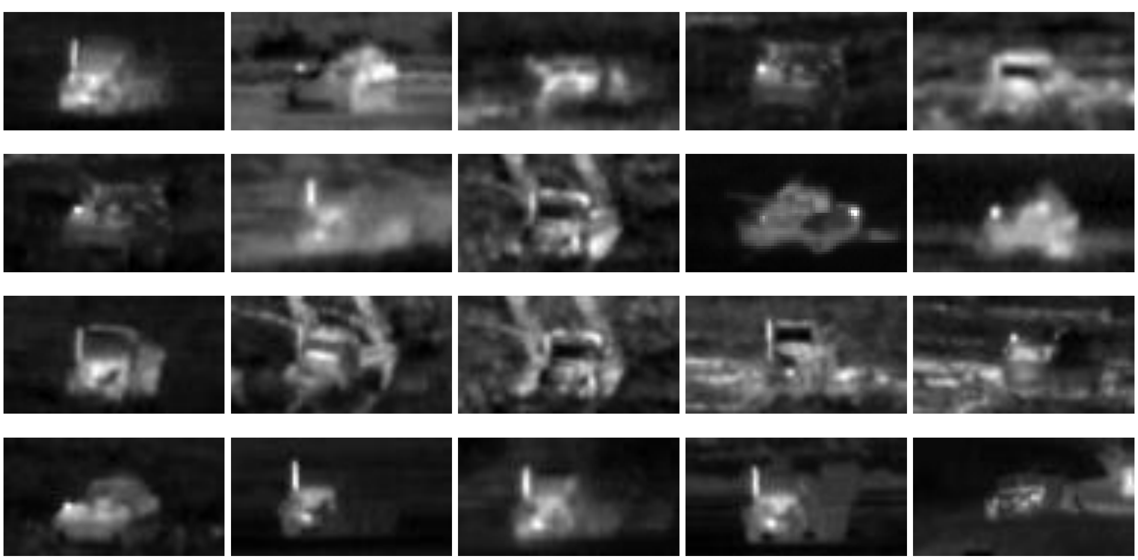


Figure 8.3.2. Middle 25 images of the third dimension of Kernel PCA

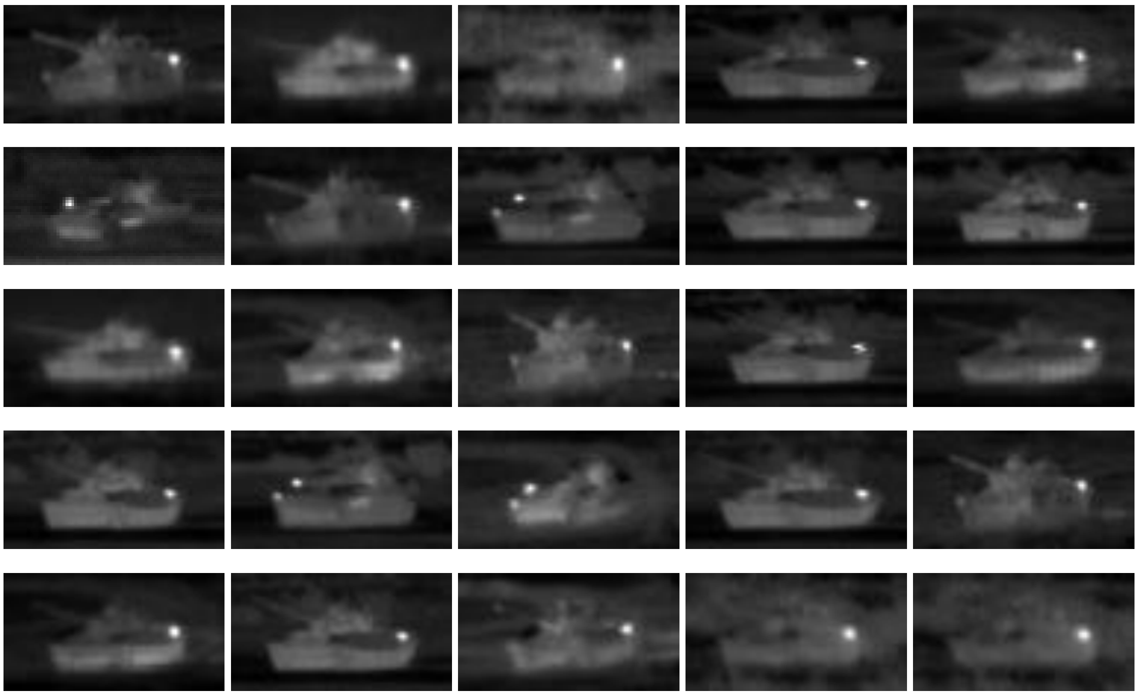
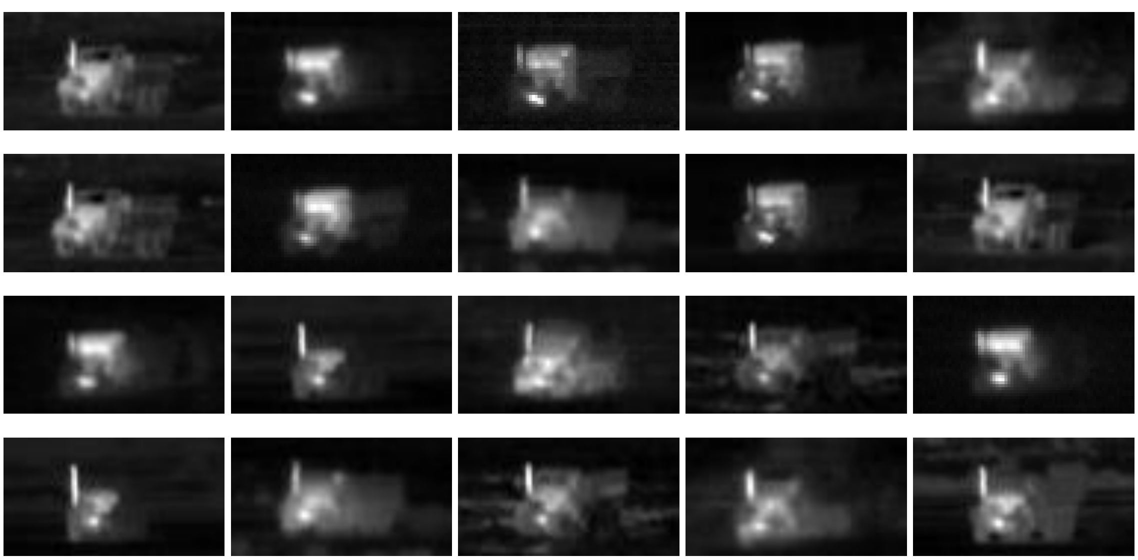


Figure 8.3.3. Last 25 images of the third dimension of Kernel PCA



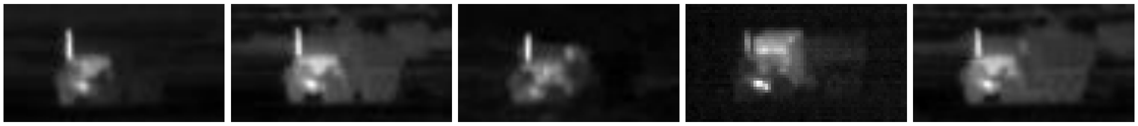


Figure 8.4.1. First 25 images of the fourth dimension of Kernel PCA

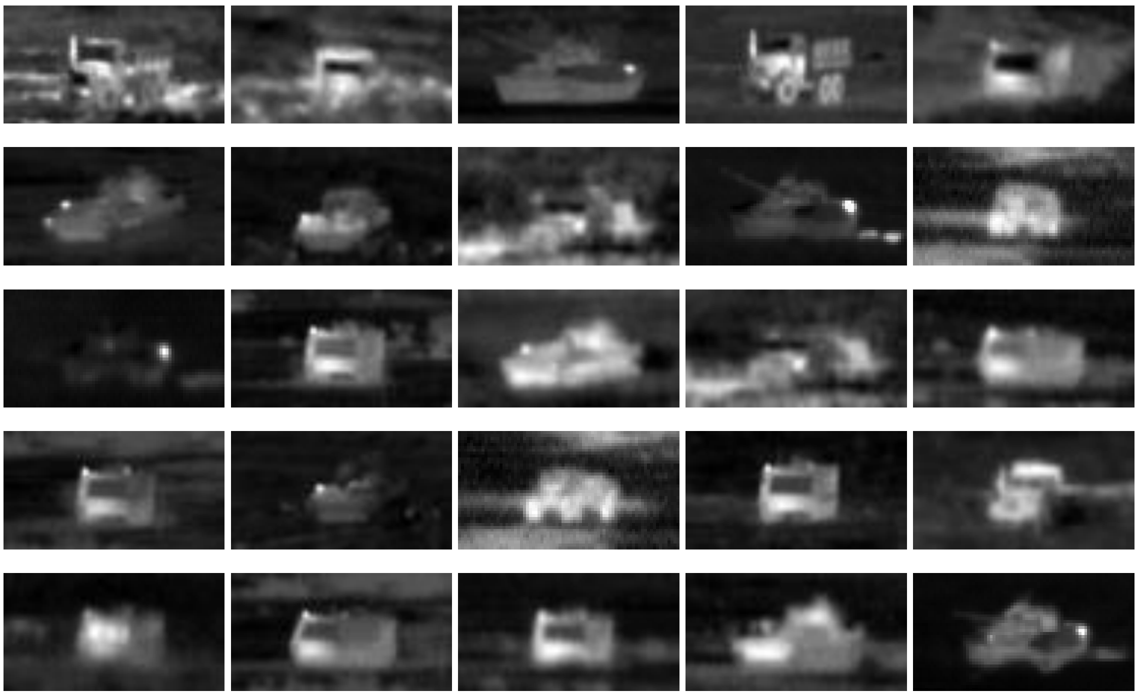


Figure 8.4.2. Middle 25 images of the fourth dimension of Kernel PCA

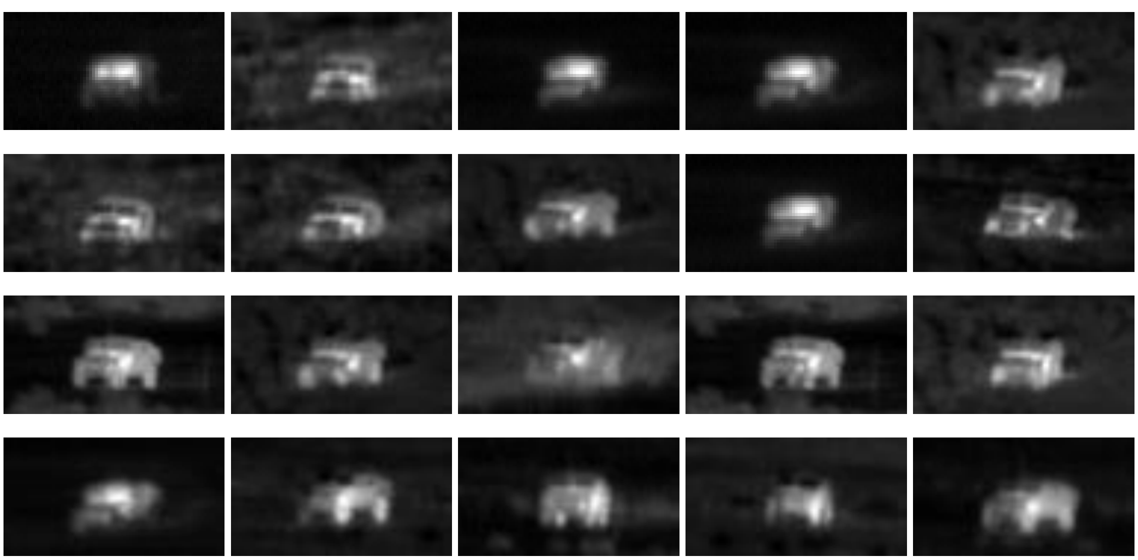
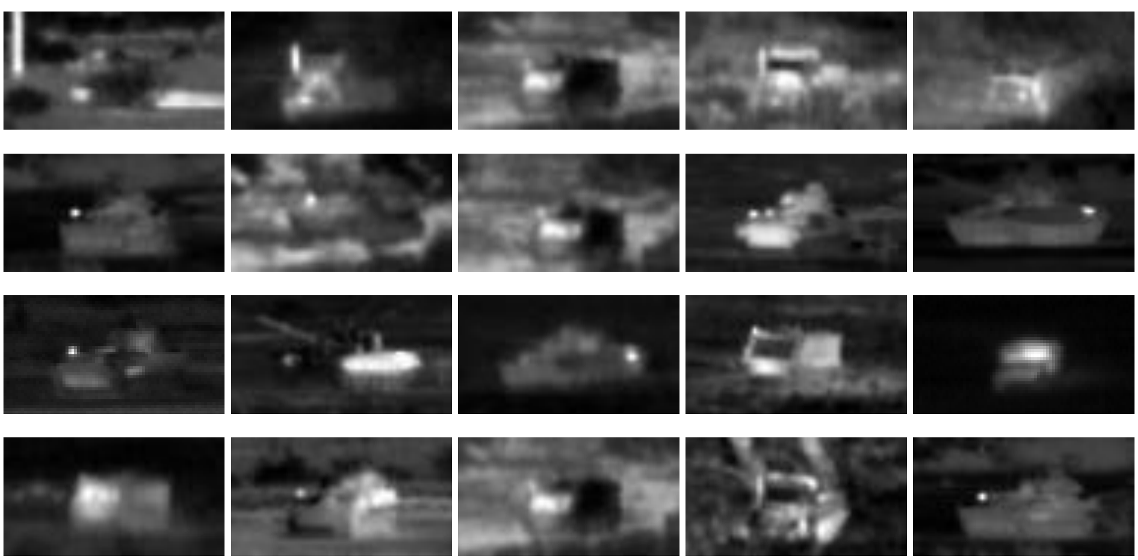


Figure 8.4.3. Last 25 images of the fourth dimension of Kernel PCA

Figure 8.5.1. First 25 images of the fifth dimension of Kernel PCA



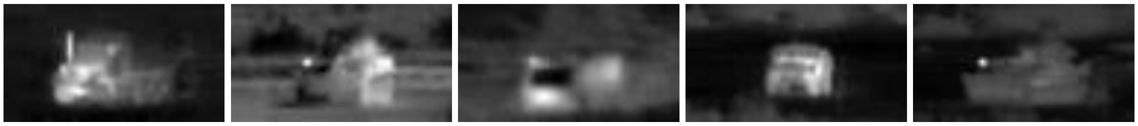


Figure 8.5.2. Middle 25 images of the fifth dimension of Kernel PCA

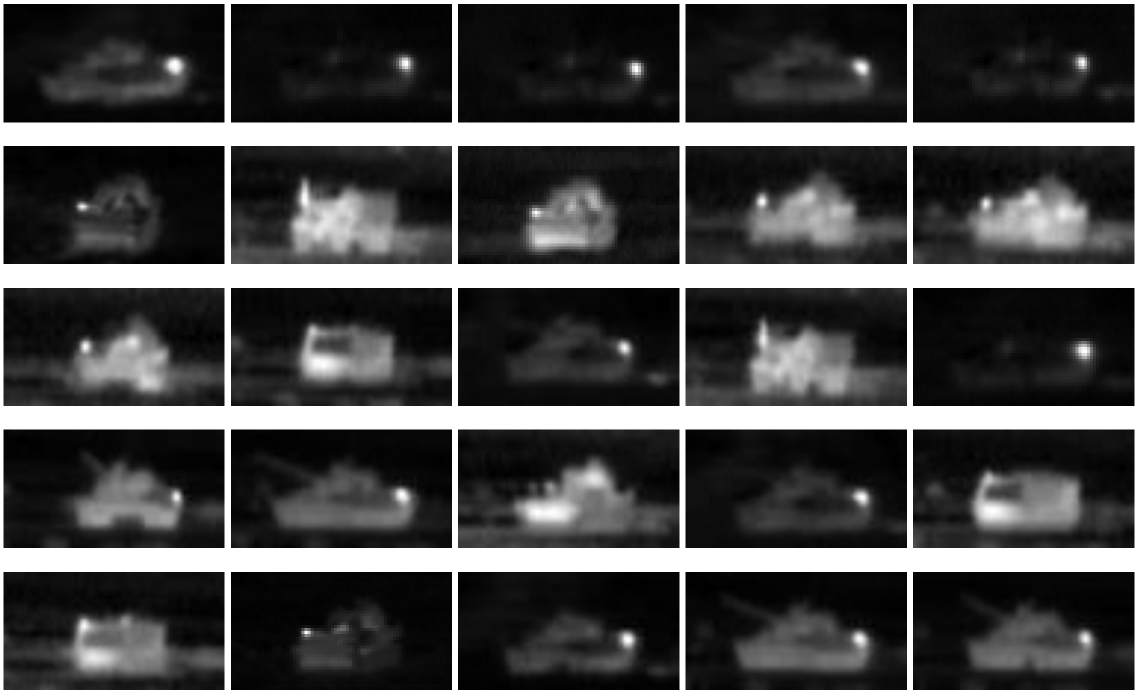
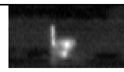
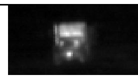
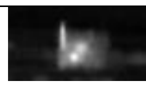
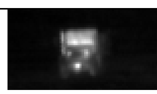
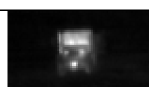
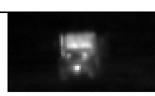
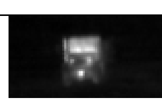


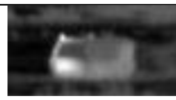
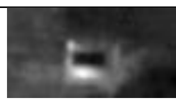
Figure 8.5.3. Last 25 images of the fifth dimension of Kernel PCA

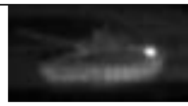
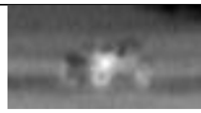
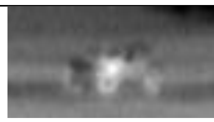
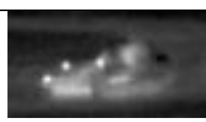
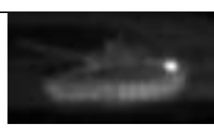
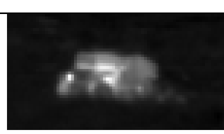
Extreme Testing Results

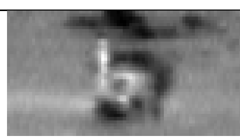
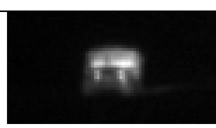
PCA

Title	Photos	First Dimension Position	Second Dimension Position	Third Dimension Position	Fourth Dimension Position	Fifth Dimension Position
Color Background		36	0	128	241	683
		312	639	617	637	655

Color of Vehicle		69	1	30	578	344
		89	639	250	508	388
Exhaust Color		418	320	349	531	181
		662	309	204	356	71
Exhaust Present		1	214	684	680	34
		549	550	481	391	181
Headlight Color		94	257	614	455	411
		662	310	204	352	72
Headlight Number		187	22	284	103	204
		662	309	204	343	73
Headlight Shape		527	326	222	251	111
		662	309	202	348	71
Headlight Size		352	333	51	503	157
		662	309	203	357	71

Shape		92	158	649	135	447
		204	562	385	226	415
Shape of Back		167	660	684	605	624
		371	566	126	274	645
Shape of Front		69	253	662	37	403
		171	300	369	632	408
Size		43	493	18	11	10
		153	61	654	71	188
Size Length		161	679	640	650	407
		366	626	565	193	473
Size Width		43	493	17	11	10
		92	176	180	515	52
		319	319	550	264	612

Weapon Color		462	153	492	292	657
Weapon Number		37	322	581	233	78
		194	454	111	557	522
Weapon Presents		37	326	578	239	76
		93	158	647	136	455
Weapon Shape		320	319	550	271	611
		193	454	116	556	525
Weapon Size		168	488	453	555	631
		462	154	493	299	658
Wheel Color		591	362	185	283	156
		43	493	17	11	10

Wheel Number		15	248	685	520	16
		152	61	655	79	169
Wheel Shape		15	247	685	547	18
		68	541	43	6	666
Wheel Size		75	278	57	154	12
		92	175	179	530	51
Window Color		18	98	279	457	50
		688	216	344	155	135
Window Number		85	276	437	326	202
		297	454	84	198	475

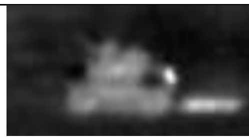
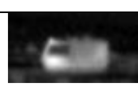
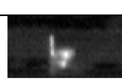
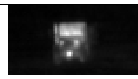
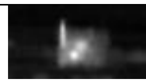
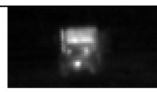
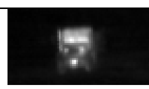
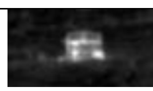
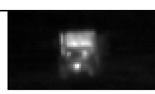
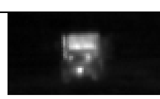
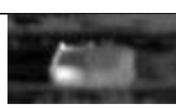
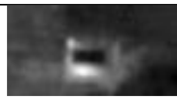
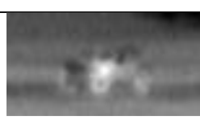
Window Shape		282	533	486	536	299
		27	114	0	685	654
Window Size		94	257	614	455	411
		446	420	434	114	258

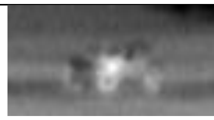
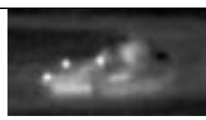
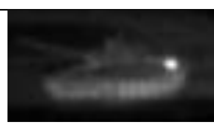
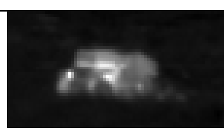
Figure 9.1: Extreme test for PCA. The table contains the title of all 27 test followed by the two images that were used for the extremes. Then the next five numbers were the positions these images appeared in each dimension.

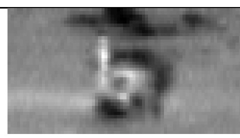
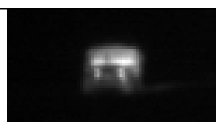
MDS

Title	Photos	First Dimension Position	Second Dimension Position	Third Dimension Position	Fourth Dimension Position	Fifth Dimension Position
Color Background		399	32	9	409	4
		532	636	352	209	101
Color of Vehicle		487	28	32	377	31
		375	377	587	539	47
		322	453	101	157	186

Exhaust Color		194	513	176	3	354
Exhaust Present		680	42	49	659	0
		181	651	220	152	307
Headlight Color		623	159	109	541	54
		165	516	185	2	376
Headlight Number		615	53	90	381	228
		181	523	185	2	361
Headlight Shape		207	457	147	62	255
		390	518	186	2	344
Headlight Size		179	339	197	75	103
		410	520	188	2	354
Shape		634	88	56	539	74
		264	412	333	290	66
		582	601	170	418	33

Shape of Back		290	359	535	28	152
Shape of Front		649	122	91	582	48
		275	230	146	411	85
Size		634	99	574	491	9
		474	101	10	499	175
Size Length		125	635	321	456	34
		493	636	260	192	140
Size Width		631	102	558	486	9
		243	75	146	481	55
Weapon Color		602	268	256	184	199
		541	199	174	108	340
		572	85	298	637	35

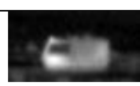
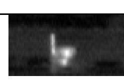
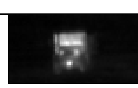
Weapon Number		314	180	451	154	84
Weapon Presents		570	87	297	632	35
		486	93	40	488	57
Weapon Shape		466	240	166	190	181
		553	201	484	183	80
Weapon Size		579	344	390	434	80
		536	190	175	107	360
Wheel Color		145	478	223	25	332
		641	90	578	511	8
Wheel Number		654	65	23	654	3
		488	106	12	489	167

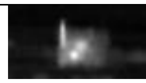
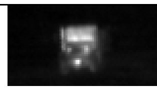
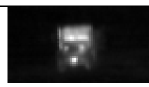
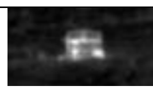
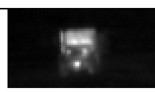
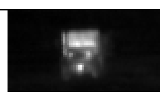
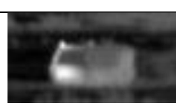
Wheel Shape		234	99	22	640	0
		664	128	627	468	35
Wheel Size		562	65	424	491	33
		246	80	151	478	54
Window Color		654	34	261	651	18
		229	584	105	6	575
Window Number		338	118	171	552	50
		474	231	543	133	229
Window Shape		495	518	241	207	114
		305	72	254	482	0

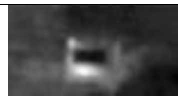
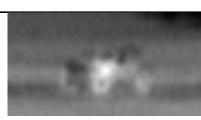
Window Size		404	163	98	532	48
		551	522	164	255	299

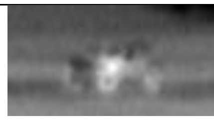
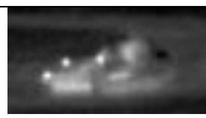
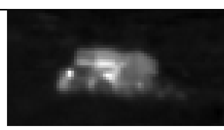
Figure 9.2: Extreme test for MDS. The table contains the title of all 27 test followed by the two images that were used for the extremes. Then the next five numbers were the positions these images appeared in each dimension.

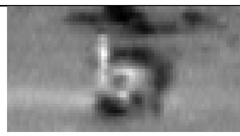
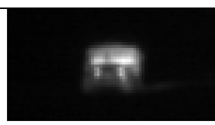
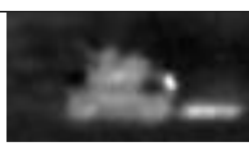
Isomap

Title	Photos	First Dimension Position	Second Dimension Position	Third Dimension Position	Fourth Dimension Position	Fifth Dimension Position
Color Background		23	538	262	667	290
		381	79	446	6	273
Color of Vehicle		47	217	280	664	58
		92	671	156	310	70
Exhaust Color		548	556	67	352	222
		683	342	73	671	362
		1	217	540	190	575

Exhaust Present		618	533	568	416	441
Headlight Color		106	391	405	282	203
		683	342	72	671	358
Headlight Number		170	42	499	339	447
		683	343	70	671	359
Headlight Shape		595	451	269	676	533
		683	341	72	670	350
Headlight Size		495	354	142	633	205
		683	340	69	669	369
Shape		81	348	630	412	576
		500	509	674	10	188
Shape of Back		202	637	552	0	190
		468	538	350	462	236

Shape of Front		51	457	246	287	666
		132	315	202	240	301
Size		73	653	658	613	30
		119	162	437	413	667
Size Length		488	14	179	606	224
		83	101	80	221	115
Size Width		73	533	77	415	54
		74	653	654	614	26
Weapon Color		351	232	669	288	315
		295	101	601	346	256
Weapon Number		34	370	298	429	503
		229	377	311	29	40

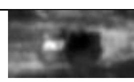
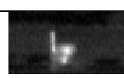
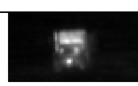
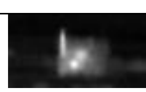
Weapon Presents		33	370	295	431	503
		87	311	636	408	594
Weapon Shape		350	235	669	289	316
		224	384	326	36	40
Weapon Size		148	389	382	87	125
		294	104	602	345	253
Wheel Color		651	406	206	642	566
		73	653	654	614	29
Wheel Number		17	280	230	179	542
		119	164	440	424	670
Wheel Shape		17	262	581	103	576

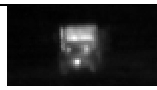
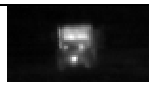
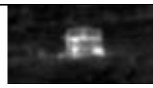
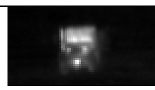
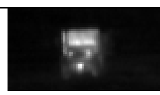
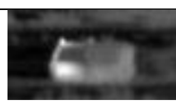
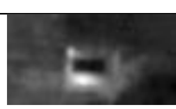
		99	539	13	389	32
Wheel Size		50	581	291	409	54
		73	525	79	386	50
Window Color		672	343	212	663	589
		8	191	153	341	447
Window Number		71	358	337	517	487
		221	268	660	481	289
Window Shape		252	405	537	318	451
		69	29	7	618	69
Window Size		583	299	312	420	531

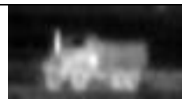
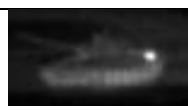
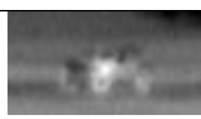
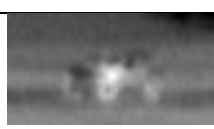
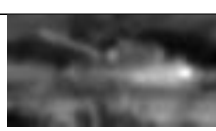
		134	293	134	365	166

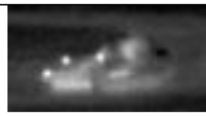
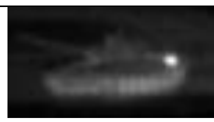
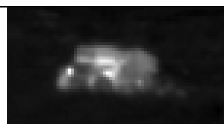
Figure 9.3: Extreme test for Isomap. The table contains the title of all 27 test followed by the two images that were used for the extremes. Then the next five numbers were the positions these images appeared in each dimension.

Kernel PCA

Title	Photos	First Dimension Position	Second Dimension Position	Third Dimension Position	Fourth Dimension Position	Fifth Dimension Position
Color Background		516	578	155	230	251
		454	148	450	351	6
Color of Vehicle		545	609	200	173	324
		688	430	270	221	454
Exhaust Color		264	184	357	120	339
		212	415	34	129	237
Exhaust Present		516	569	108	202	501
		240	59	53	512	566
		633	496	332	405	308

Headlight Color		212	414	33	128	237
Headlight Number		452	577	497	440	33
		212	406	33	128	237
Headlight Shape		188	207	53	538	47
		210	405	33	127	236
Headlight Size		354	233	147	181	37
		210	413	33	129	241
Shape		609	538	322	458	298
		487	270	489	314	441
Shape of Back		544	369	269	437	74
		385	142	410	323	327
Shape of Front		666	572	241	426	359
		487	334	506	210	297

Size		640	603	133	172	407
		489	569	475	542	138
Size Length		580	339	251	237	62
		402	73	264	616	84
Size Width		639	605	137	174	410
		641	518	293	160	195
Weapon Color		321	271	655	546	274
		128	440	684	553	201
Weapon Number		688	630	168	236	418
		476	284	492	132	178
Weapon Presents		688	630	167	235	415
		609	535	332	458	298

Weapon Shape		321	263	652	547	392
		477	285	489	132	179
Weapon Size		519	262	563	437	331
		129	442	684	552	478
Wheel Color		143	227	42	341	55
		640	604	130	172	409
Wheel Number		545	609	108	191	508
		489	569	476	542	138
Wheel Shape		544	612	116	192	506
		654	539	223	282	361
Wheel Size		670	575	207	154	263

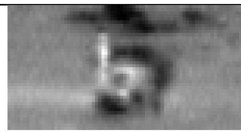
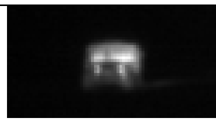
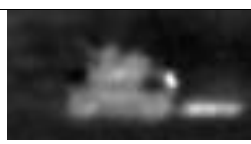
		644	519	293	159	195
Window Color		625	637	127	148	482
		144	561	20	604	235
Window Number		670	523	290	323	255
		369	144	530	393	47
Window Shape		417	152	397	329	493
		534	578	132	173	457
Window Size		633	495	330	404	308
		227	169	454	581	439

Figure 9.4: Extreme test for Kernel PCA. The table contains the title of all 27 test followed by the two images that were used for the extremes. Then the next five numbers were the positions these images appeared in each dimension.